Parameterized verification of Broadcast networks of Register automata

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ANR PaVeDyS

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   - Basic model
   - With registers

2. Signature BNRA
   - Well quasi-orders
   - Decidability proof

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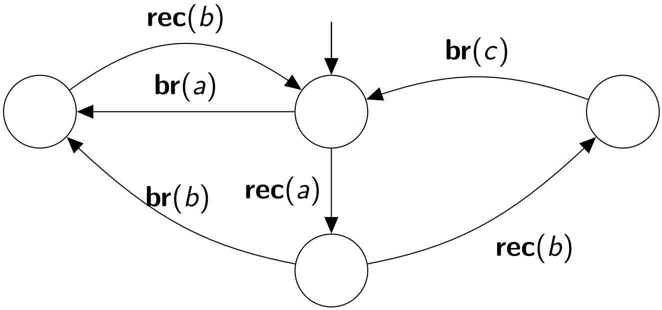

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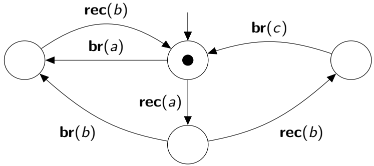

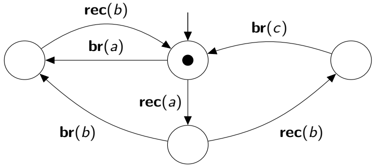

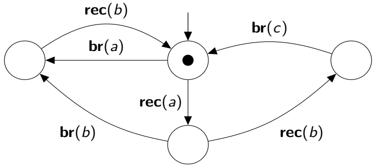

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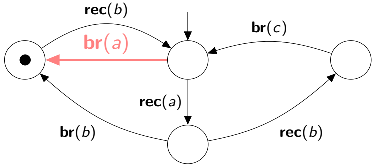

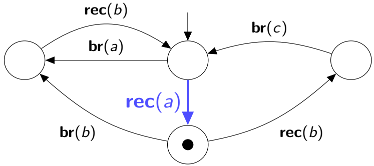

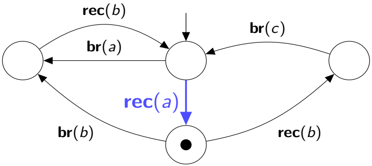

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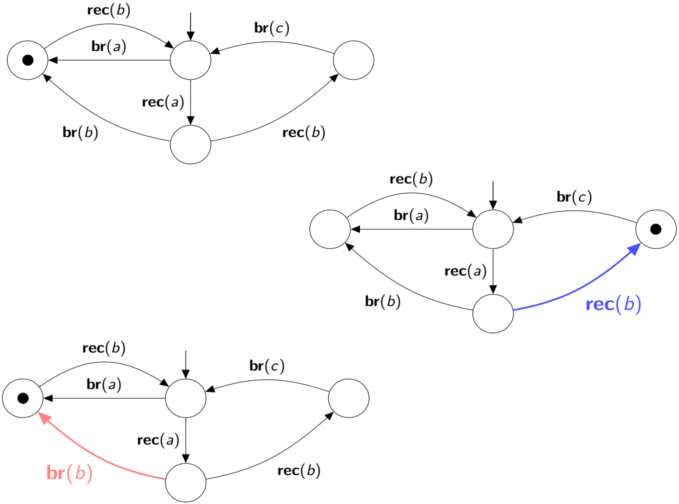

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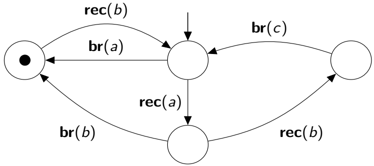

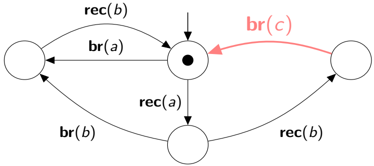

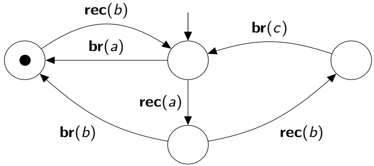

Broadcast Networks

Definition[1]

(Reconfigurable) Broadcast Network $= (Q, M, \Delta, q_0)$ with $\Delta \subseteq Q \times \{\mathbf{br}(m), \mathbf{rec}(m) \mid m \in M\} \times Q$.

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Problems

COVER: Is there a run in which **an** agent reaches q_f?
TARGET: Is there a run in which **all agents** reach q_f **simultaneously**?

Both problems are decidable in PTIME[1,2].

[1] Delzanno, Sangnier, Zavattaro, CONCUR'10
[2] Fournier, PhD thesis, 2015

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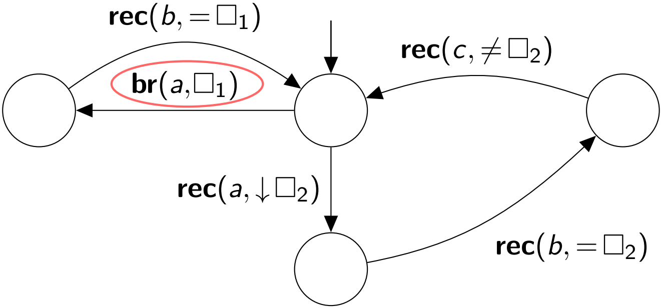

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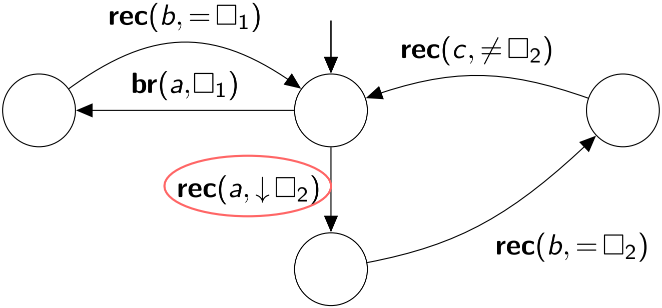

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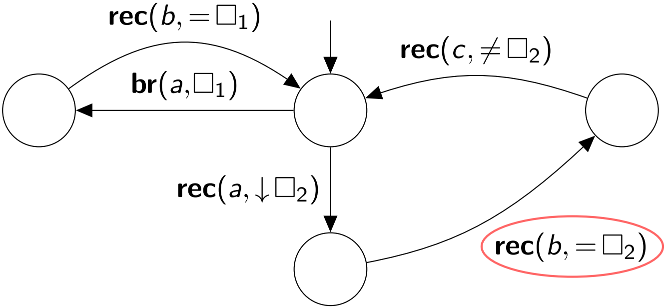

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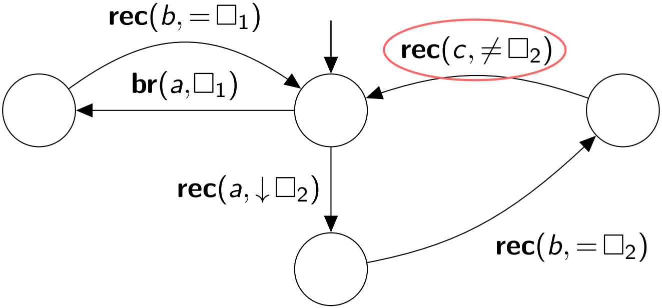

Broadcast Networks of Register Automata (BNRA)[3]

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Remark: the model where one allows to send two messages per broadcast is undecidable[3].

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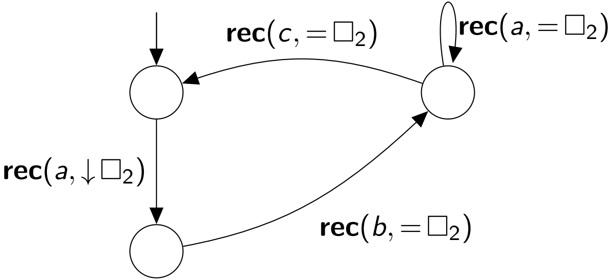
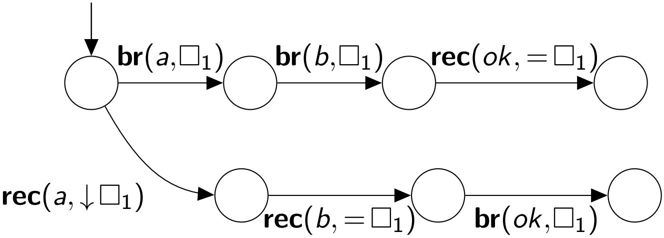
Things we can do

We can check that a sequence of messages all come from the same agent.



Things we can do

We can check that a sequence of messages we sent was received.



Parameterized verification principles

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Copycat principle

Given a run ρ, we can construct a run made of many copies of ρ running in parallel.

Main theorem

COVER is decidable for BNRA.

Signature BNRA

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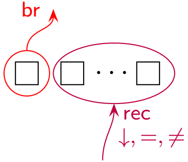
An agent never modifies its first register, and only broadcasts with the value of its first signature.
Other registers are used to store and compare values received.

The first register acts as an identity with which agents sign their messages.



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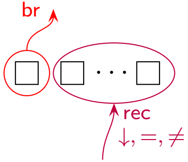
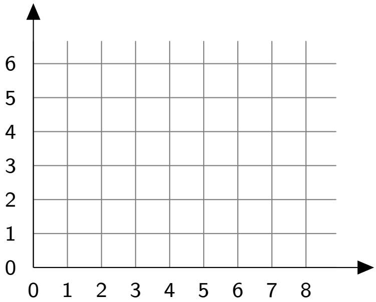


Messages received with the same value come from the same agent.

Well quasi-orders



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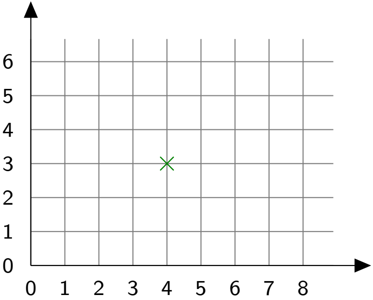

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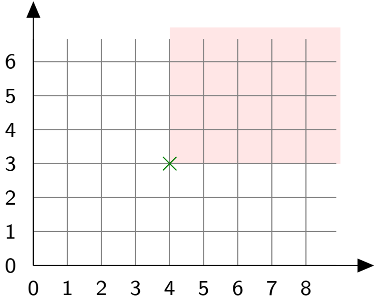

$(4, 3)$

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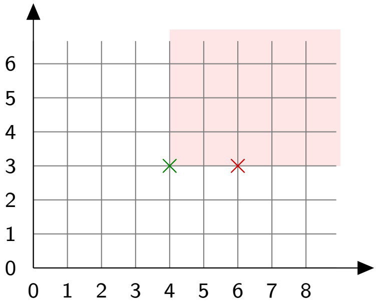

$(4,3) \rightarrow (6,3)$

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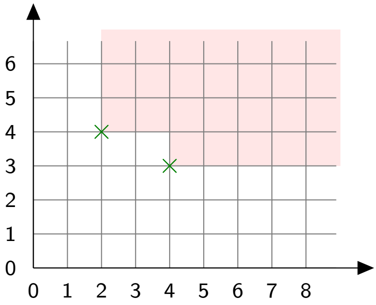

$(4,3) \to (2,4)$

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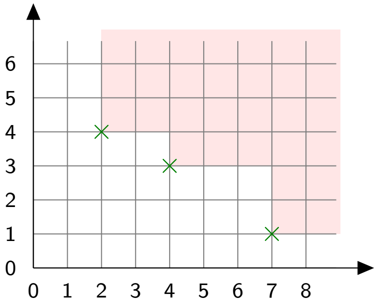

$(4, 3) \rightarrow (2, 4) \rightarrow (7, 1)$

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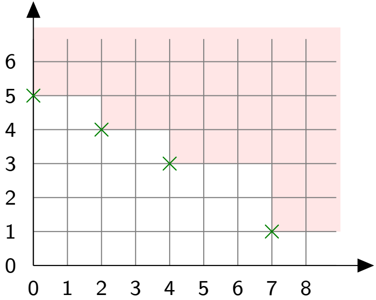

$(4,3) \rightarrow (2,4) \rightarrow (7,1) \rightarrow (0,5)$

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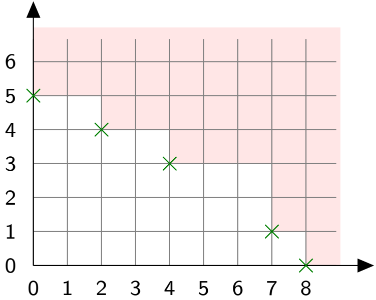

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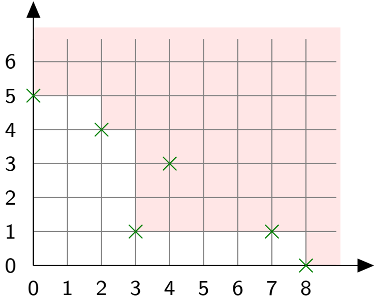

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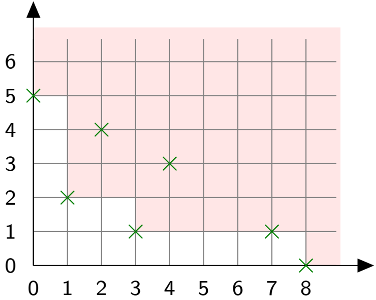

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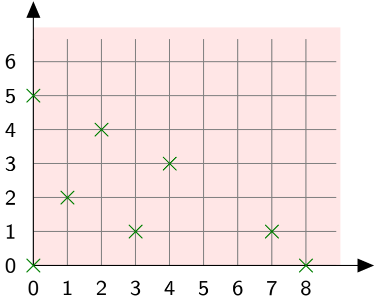

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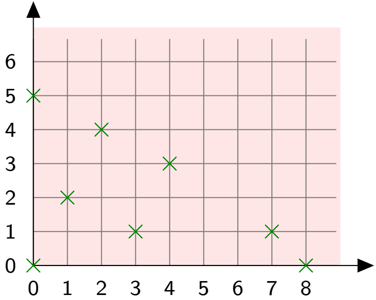

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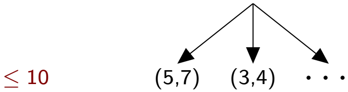

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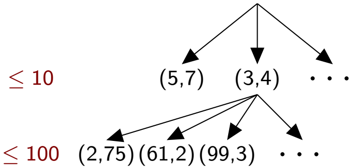

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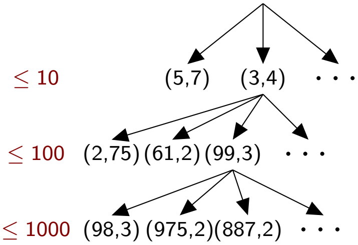

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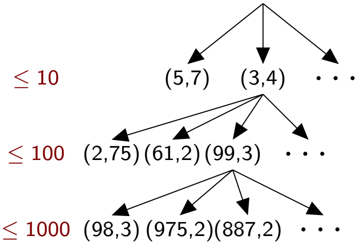

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In fact, there is a computable bound on the length of the longest branch.

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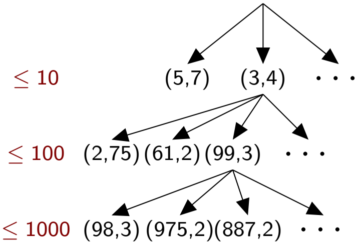

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We can enumerate all possible such trees!

Well quasi-orders: Subwords

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For all finite alphabet Σ, the subword order $\preceq$ is a well quasi-order over Σ^*.

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$\Leftrightarrow$ Any sequence $w_0, w_1, w_2, \ldots$ of words over Σ such that $w_i \not\preceq w_j$ for all $i < j$ is **finite**.

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Given a finite alphabet Σ and a computable function $B : \mathbb{N} \to \mathbb{N}$, the set of sequences $(w_i)_{i\in\mathbb{N}}$ over Σ such that

- $w_i \not\preceq w_j$ for all $i < j$
- $|w_i| \leq B(i)$ for all i

is finite and computable.

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Towards a tree abstraction

Assume that there is a valid run ρ for COVER.

Observation 1

If agent a broadcasts to agents b and c, we can make copy agent a so that b and c receive messages from distinct agents.

We can modify ρ so that each agent sends messages to one single agent.

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Observation 2

If a broadcasts m_1 to b then b broadcasts m_2 to a, we can make a copy a' of a that broadcasts to b and then stops; $a' \to b \to a$.

More generally, we can guarantee that the graph of "who sends messages to whom" has no cycle: it's a tree (or a forest) !

Tree unfoldings

$$\xrightarrow[\mathbf{rec}(m_1, v_1)\mathbf{rec}(m_2, v_2)\mathbf{rec}(m_3, v_1)]{\mathbf{br}(m_0, v_0) \qquad\qquad\qquad\qquad\qquad\qquad \mathbf{br}(m, v_0)}$$

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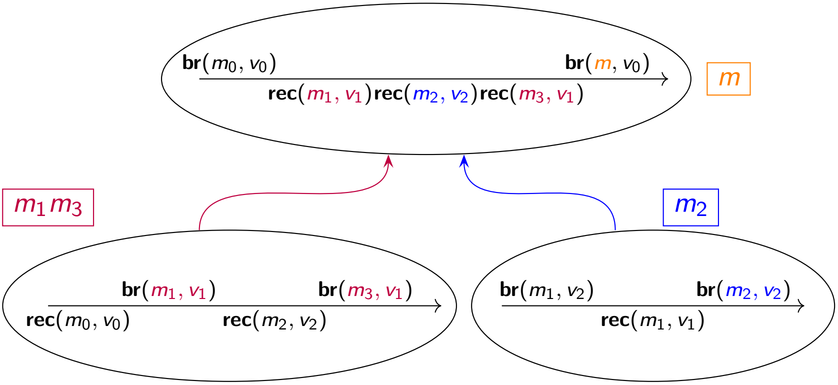

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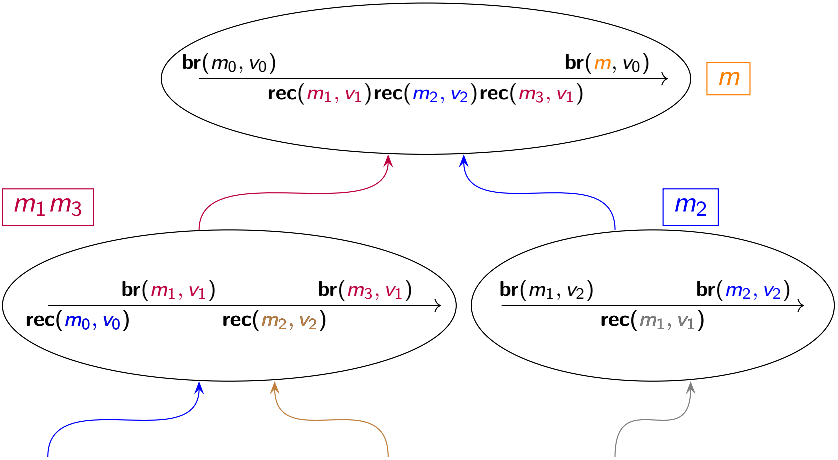

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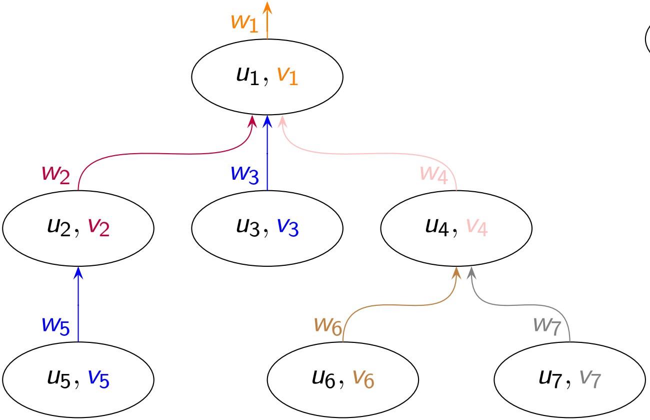
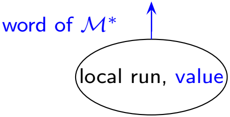
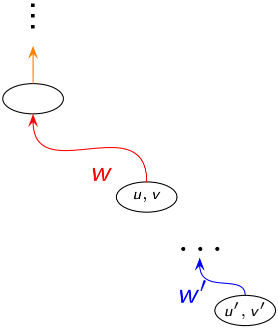




Branch reduction

Lemma

If a node labelled w has a descendant labelled w' with w a subword of w' then the tree can be reduced.



$w \preceq w'$

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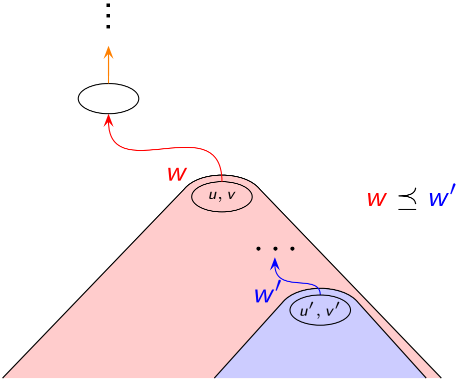

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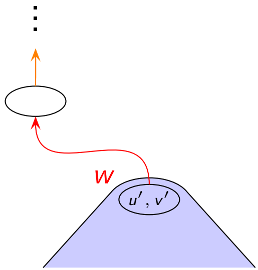

$w \preceq w'$

Branch reduction

- We can assume that a node labelled w has no descendant labelled $w' \succeq w$.

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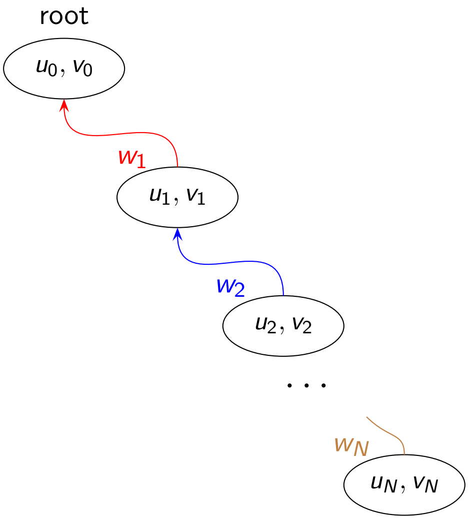
Shortening long local runs

There exists a primitive recursive function $\varphi : \mathbb{N} \to \mathbb{N}$ such that, if an agent must broadcast k messages, its local run does not need to have more than $k\,\varphi(|\mathcal{P}|)$ steps.

$|\mathcal{P}|$: size of the protocol
Proof by shortening arguments (a bit involved)

Bounding the branches



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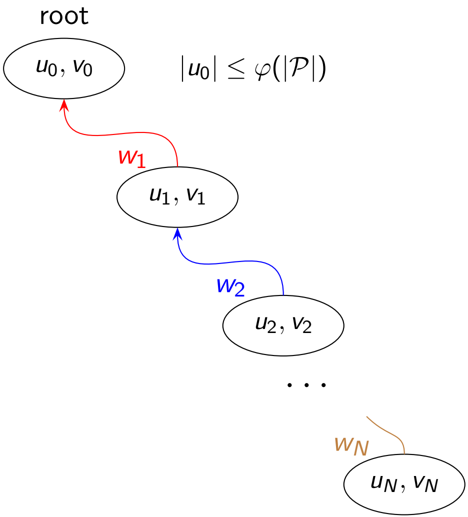

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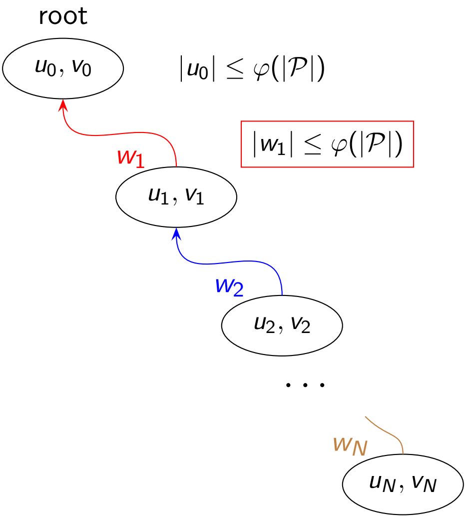

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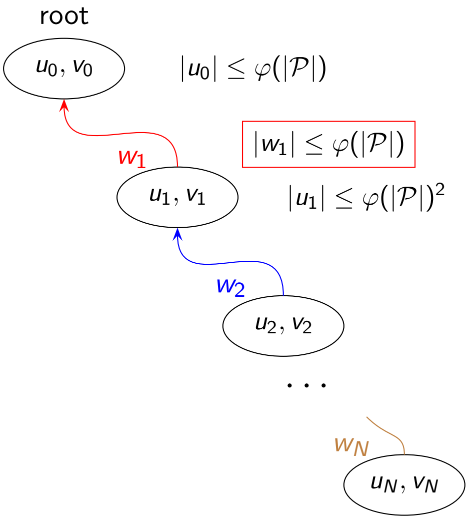

$|u_0| \leq \varphi(|\mathcal{P}|)$

$\boxed{|w_1| \leq \varphi(|\mathcal{P}|)}$

$|u_1| \leq \varphi(|\mathcal{P}|)^2$

Bounding the branches

root

u_0, v_0 $\qquad |u_0| \leq \varphi(|\mathcal{P}|)$

w_1 $\qquad \boxed{|w_1| \leq \varphi(|\mathcal{P}|)}$

u_1, v_1 $\qquad |u_1| \leq \varphi(|\mathcal{P}|)^2$

w_2 $\qquad \boxed{|w_2| \leq \varphi(|\mathcal{P}|)^2}$

u_2, v_2 $\qquad |u_2| \leq \varphi(|\mathcal{P}|)^3$

$\cdots$

w_N $\qquad \boxed{|w_N| \leq \varphi(|\mathcal{P}|)^N}$

u_N, v_N $\qquad |u_N| \leq \varphi(|\mathcal{P}|)^{N+1}$

Decidability and complexity

Bounds

We use the previous argument to bound (in an irreducible tree):

- the length of all branches,
- the size of every node,
- the maximal degree of the tree.

This bounds the total space needed to store such a tree.

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Can be extended to the non-signature case.

Complexity lower bounds

Lossy Channel Systems

A *Lossy Channel System* (LCS) is a transition system with a FIFO queue and unreliable writes.

Theorem

LCS reachability is $\mathbf{F}_{\omega^\omega}$-hard[a].

[a] Schnoebelen, Information Processing Letters '08

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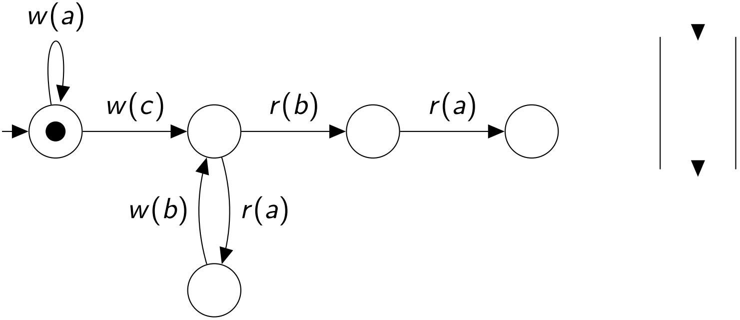
It is however NP-complete when each agent has only one register.

Conclusion

Thank you for your attention!

Complexity: encoding Lossy Channel Systems

Lossy Channel System = Transition system with FIFO memory + unreliable writes.



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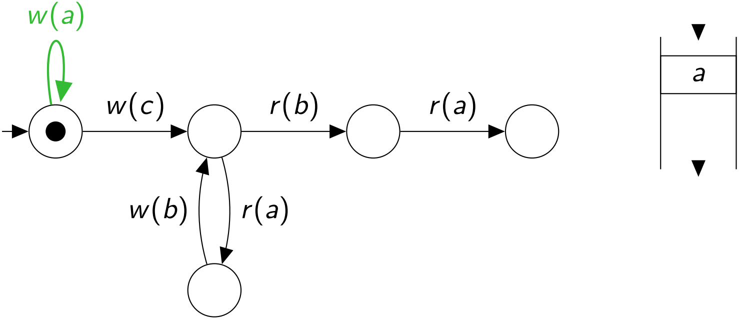

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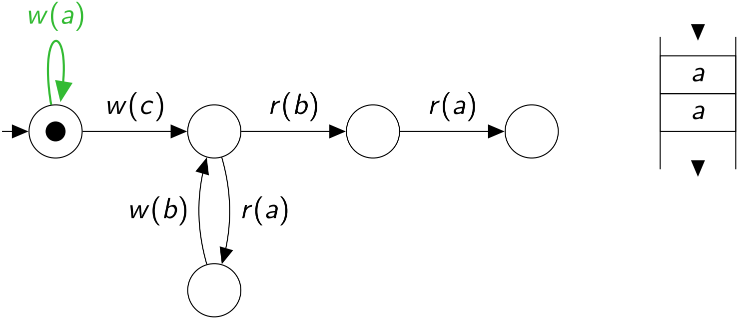

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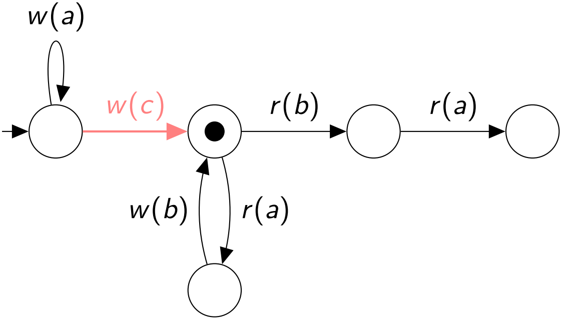

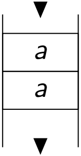

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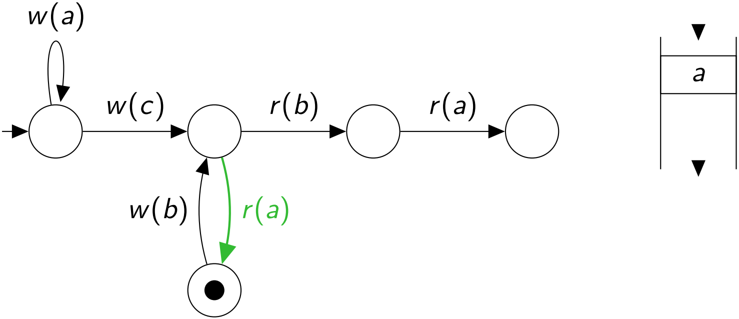

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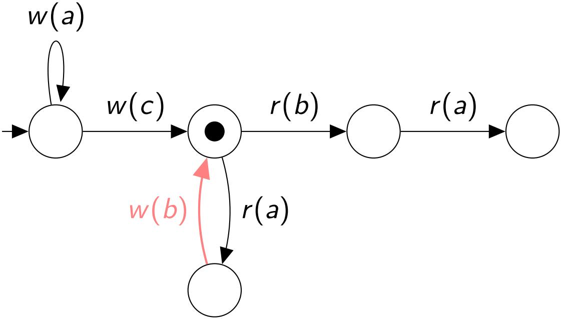

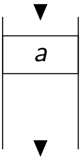

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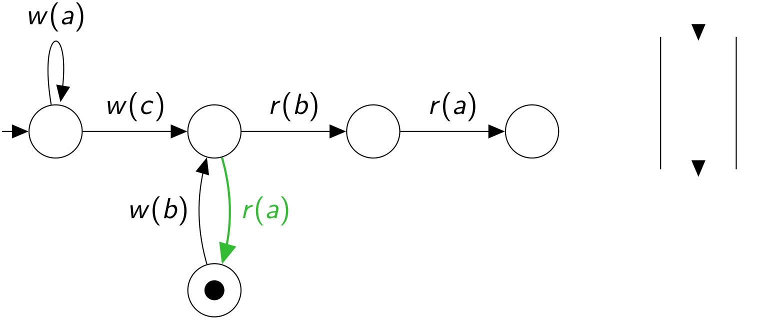

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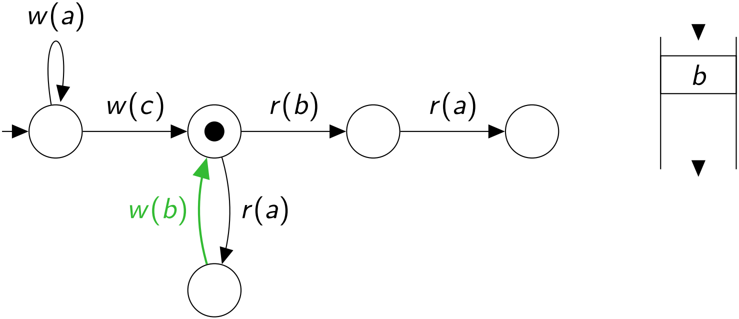

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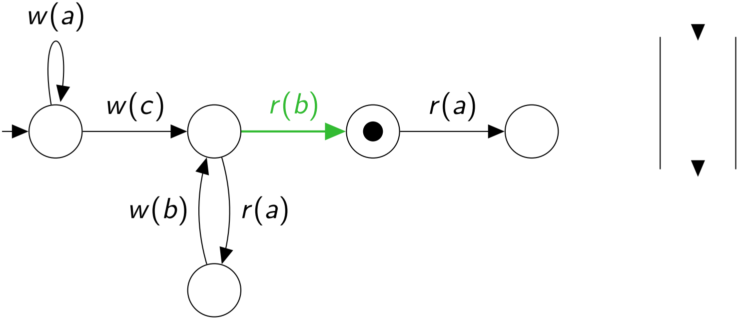

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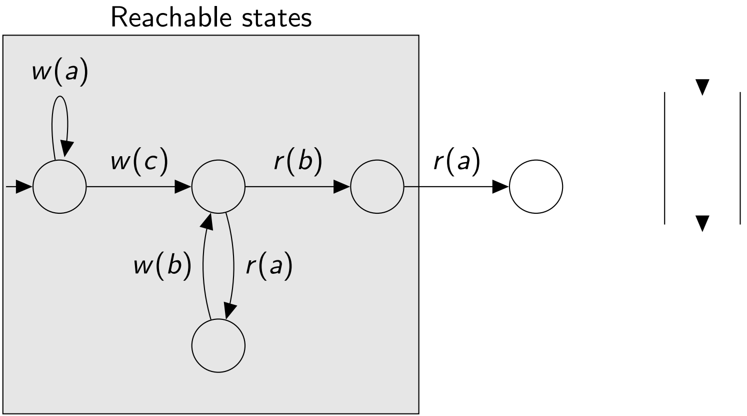
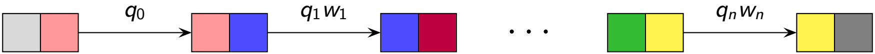


Complexity: encoding Lossy Channel Systems

We simulate an LCS through a chain of agents that each apply a transition.

Each agent stores:

- An identifier for itself
- Its predecessor's identifier



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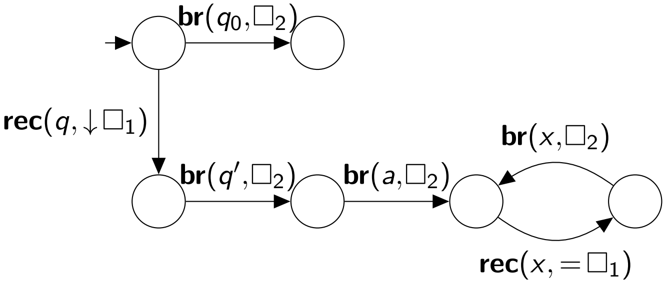

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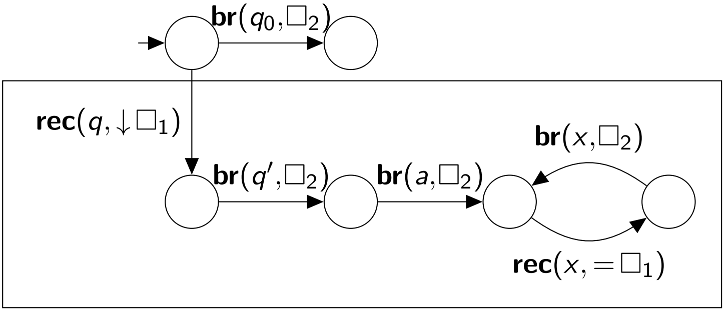

For each transition $q \xrightarrow{w(a)} q'$ of the LCS

Complexity results

$\mathbf{F}_{\omega^\omega}$ = Hyper-Ackermannian complexity class.

Theorem

LCS reachability is $\mathbf{F}_{\omega^\omega}$-hard[a].

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Theorem

Cover in BNRA with one register is **NP**-complete.