Modeling and Verifying Randomized Fault-Tolerant Distributed Algorithms

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DisCoTec - June 17th 2020

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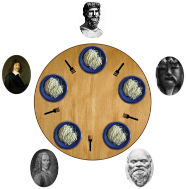
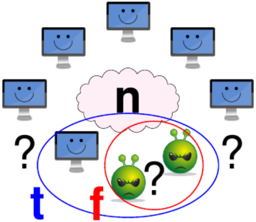
Randomization in distributed computing

- To prevent attacks
- To improve complexity
  - average complexity may only be better than worst-case
- To make impossible things possible!
  - impossibility of symmetric solution to **dining philosophers problem**
    - use randomness to break symmetry [Lehman Rabin'81]
  - impossibility of **consensus** in asynchronous setting as soon as one process can crash [Fischer Lynch Paterson'85]
    - use randomness to rule out non-terminating executions [BenOr'83]



Fault-tolerant distributed algorithms



- n processes communicate by asynchronous message broadcast
- f processes are faulty in the current execution
- t is a known upper bound on f

Resilience condition constrains parameters n, f, t ensuring correctness

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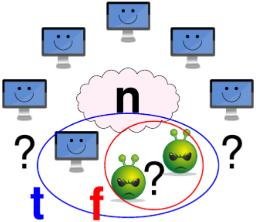

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Resilience condition constrains parameters n, f, t ensuring correctness

Ben Or's **randomized** algorithm solves consensus assuming $f \leq t < \frac{n}{5}$

The need for parameterized and automated verification

Need for **parameterized** verification

- correctness should hold **under all parameter valuations** that meet the resilience condition

$$\forall n, t, f \quad f \leq t < \frac{n}{5} \quad \Longrightarrow \quad C(n,t) || \cdots || C(n,t) || F || \cdots || F \models \varphi$$

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Need for **automated** verification

- mostly hand-written proofs in the literature
- non-determinism combined with probabilities

Proofs of correctness for probabilistic distributed systems are extremely slippery

[Lehmann Rabin'81]

Threshold automata to the rescue

for non-randomized fault-tolerant distributed algorithms

- locations represent algorithm control points
- shared variables count sent messages of each type
- guards as linear constraints on variables and parameters

[Konnov Veith Widder CAV'15, Konnov Lazić Veith Widder POPL'17]

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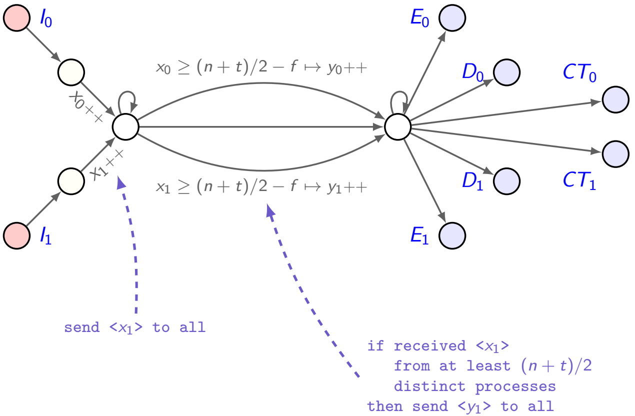

[Konnov Veith Widder CAV'15, Konnov Lazić Veith Widder POPL'17]

Semantics of threshold automata

- infinite counter system
  - finitely many counters: 1 per location of the TA
  - unbounded counter values because of parameters

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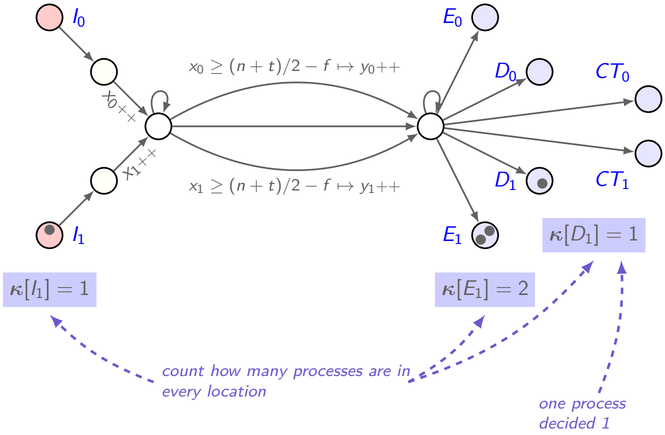

Specifying and verifying correctness

- $\mathsf{ELTL}_{\mathsf{FT}}$: LTL fragment without Next, and with counters
- atomic propositions: whether counter value is 0 or not

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- atomic propositions: whether counter value is 0 or not

Agreement: No two correct processes decide differently (safety)

$$\mathbf{F}\ \boldsymbol{\kappa}[D_v] > 0 \quad \rightarrow \quad \mathbf{G}\ \boldsymbol{\kappa}[D_{1-v}] = 0$$

Termination: Eventually all correct processes decide (liveness)

$$\mathbf{F} \bigwedge_{\ell \in \mathcal{L} \setminus \{D_0, D_1\}} \boldsymbol{\kappa}[\ell] = 0$$

Given a threshold automaton TA, a specification φ in $\mathsf{ELTL}_{\mathsf{FT}}$, and a resilience condition RC, one can decide whether for all parameters satisfying RC, Sys(TA) $\models \varphi$

Tool support: ByMC at `forsyte.at/software/bymc/`

[Konnov Veith Widder CAV'15, Konnov Lazić Veith Widder POPL'17]

Outline

How to handle randomization?

Ben Or's randomized algorithm for consensus [Ben Or'83]

```
bool v := input_value({0, 1});
int r := 1 ;
while (true) do
 send (R,r,v) to all;
 wait for n − t messages (R,r,*);
 if received (n + t) / 2 messages (R,r,w)
 then send (P,r,w,D) to all;
 else send (P,r,?) to all;
 wait for n − t messages (P,r,*);
 if received at least t + 1
    messages (P,r,w,D) then {
  v := w;                   /* enough support -> update estimate */
  if received at least (n + t) / 2
   messages (P,r,w,D)
  then decide w;                    /* strong majority -> decide */
 } else v := random(0, 1) ;      /* unclear -> coin toss      */
 r := r + 1 ;
od
```

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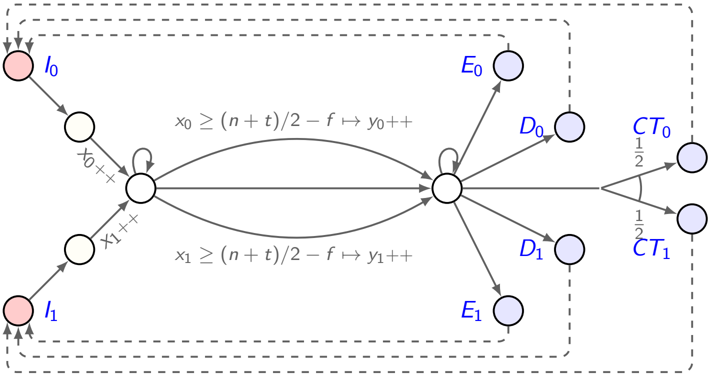
Modeling challenges

- unboundedly many rounds
- probabilistic choices for local/global coin tosses

Probabilistic threshold automata

Illustration on Ben Or's algorithm



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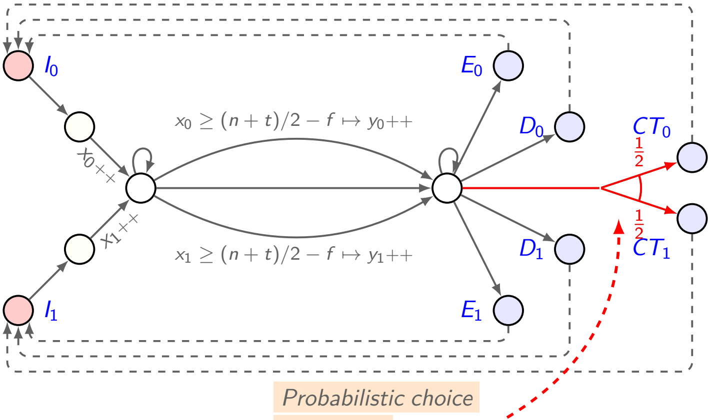

Probabilistic choice
(coin toss)

Probabilistic threshold automata

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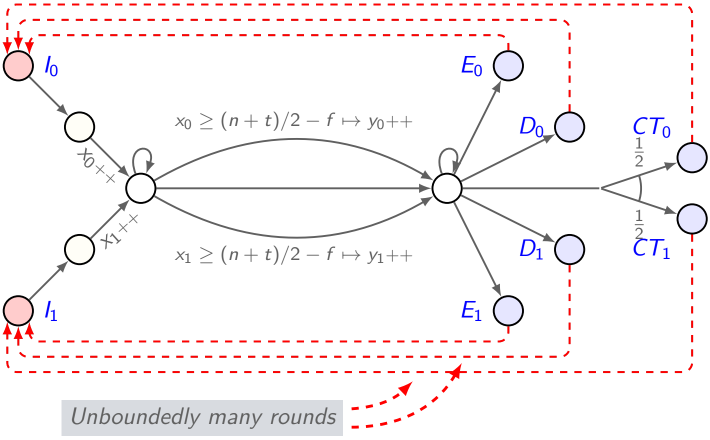

Correctness properties

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$$(\forall k \in \mathbb{N}_0)\,(\forall k' \in \mathbb{N}_0)\;\; \mathbf{A}\;\Big(\mathbf{F}\;\boldsymbol{\kappa}[D_v, k] > 0 \quad \rightarrow \quad \mathbf{G}\;\boldsymbol{\kappa}[D_{1-v}, k'] = 0\Big)$$

Validity: Any decided value was proposed initially (safety)

$$(\forall k \in \mathbb{N}_0)\;\; \mathbf{A}\;\Big(\mathbf{F}\;\boldsymbol{\kappa}[I_{1-v}, 0] = 0 \quad \rightarrow \quad \mathbf{G}\;\boldsymbol{\kappa}[D_{1-v}, k] = 0\Big)$$

Almost sure termination: under every adversary, with probability 1 every correct process eventually decides (prob. liveness)

$$\mathbb{P}_{\mathtt{a}}\,\Big(\bigvee_{k \in \mathbb{N}_0} \bigvee_{v \in \{0,1\}} \mathbf{G} \bigwedge_{\ell \in \mathcal{L} \setminus \{D_v\}} \boldsymbol{\kappa}[\ell, k] = 0\Big) = 1$$

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Verification challenges

- specifications over multiple rounds
- probabilistic guarantees

Outline

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Proving almost-sure termination
- Round-rigid adversaries
- Weak adversaries

Conclusions

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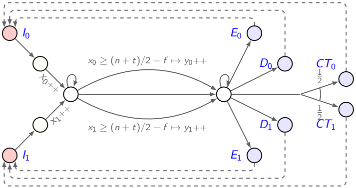

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must hold on **all** executions

$\rightarrow$ probabilistic choices can be transformed into non-determinism

Remaining challenges

- unboundedly many rounds
- specifications over multiple rounds

Solution

- reduce to one-round threshold automaton
- reduce to one-round specifications

Reduction to single-round TA

Communication-closure hyp.: only messages of current round impact
$\rightarrow$ reordering to analyze rounds in isolation

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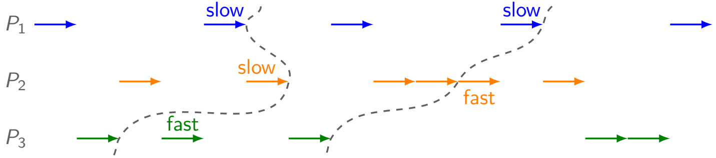

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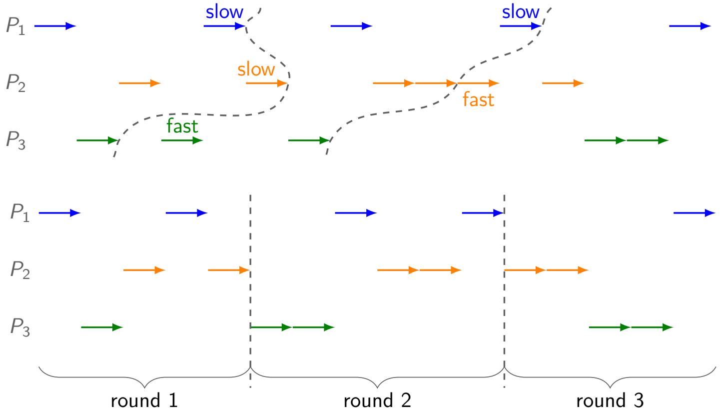

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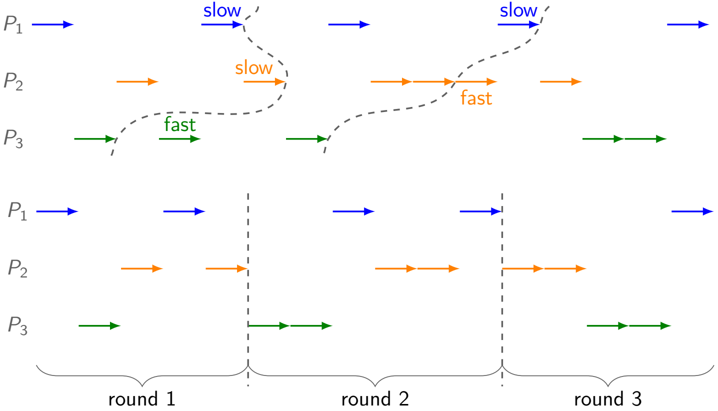
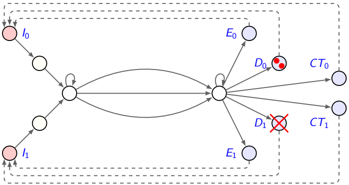


Reordering preserves validity of $\mathsf{ELTL}_{\mathsf{FT}}$ specifications.

Reduction to single-round specifications



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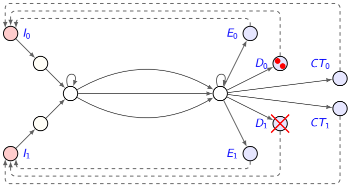

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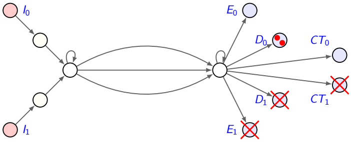

2 sufficient conditions on 1-round TA

if **F** decision v in k

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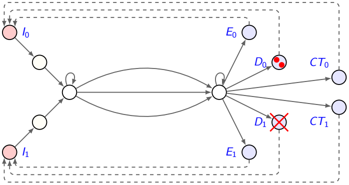

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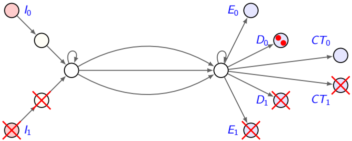

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Proving almost-sure termination
- Round-rigid adversaries
- Weak adversaries

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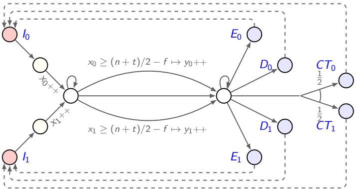

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must hold for **all** adversaries, on **almost all** executions

$\rightarrow$ probabilities matter!

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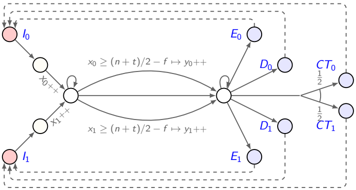

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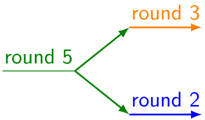

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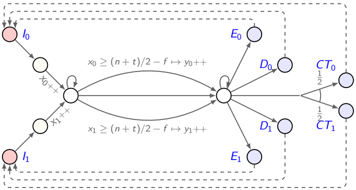

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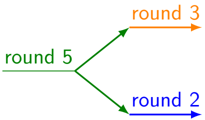

Solution

- restrict to **round-rigid** or **weak** adversaries

Round-rigid adversaries

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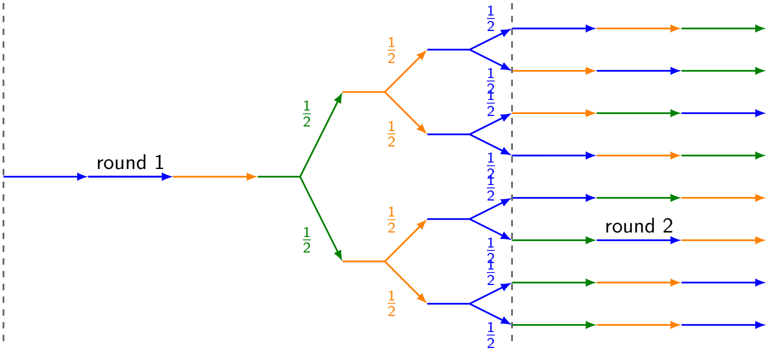
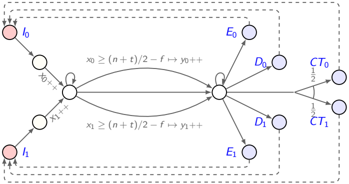


Reduction to single-round specifications

From almost-sure to being lucky



Almost-sure termination

for every **round-rigid** adv. almost surely

F all decide in k

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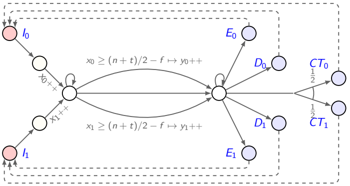

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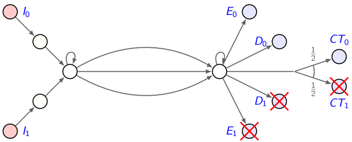

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for every adv. with bounded probability

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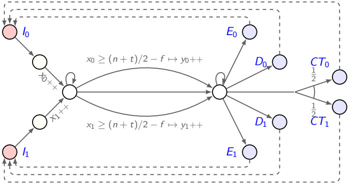

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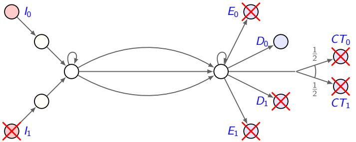

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G empty final with $1-v$ in k

Luckiness implies termination in next round

if **G** empty initial with $1-v$ in k

then **F** all decide v in k

Reduction to single-round specifications

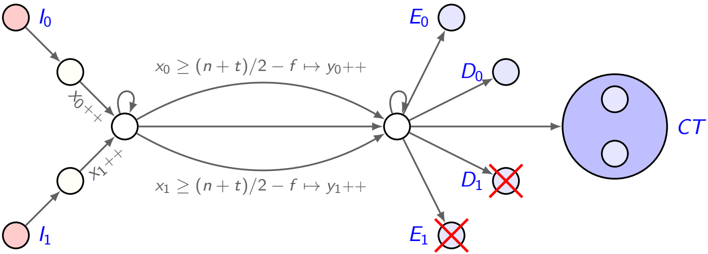
Checking that one may be lucky

for every adv. with bounded probability **G** empty final with $1-v$ in k

- the possibility of being lucky implies a bounded probability (for fixed parameter values)

  E G empty final with $1-v$ in k $\implies \mathbb{P}_{\mathsf{a}}($**G** empty final with $1-v$ in k $) \geq p > 0$

- probabilistic choices can be abstracted to obtain a TA



Experimental evaluation

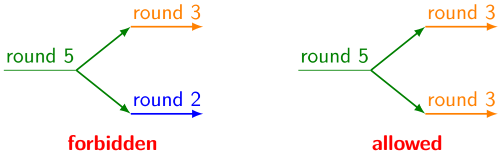
- 6 randomized consensus algorithms
- several one-round safety and liveness properties for each
- tool support: `forsyte.at/software/bymc/`

Algorithm	Verif time per property
- Ben-Or's Byzantine random. consensus	≤ 1 sec
- Ben-Or's crash random. consensus	≤ 1 sec
- Ben-Or's clean crash random. consensus	≤ 1 sec
- Bracha's randomized consensus	≤ 1 sec
- Raynal's k-set agreement	3–40 sec
- Song's and van Renesse's BOSCO	3 hours on a cluster

Weak adversaries

Weak adversary

- does not see outcome of random choices
- sees sender and type of messages, not contents
- tags messages with IDs "deliver message 42 to P1"



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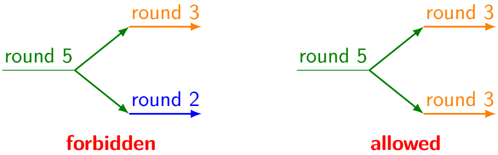
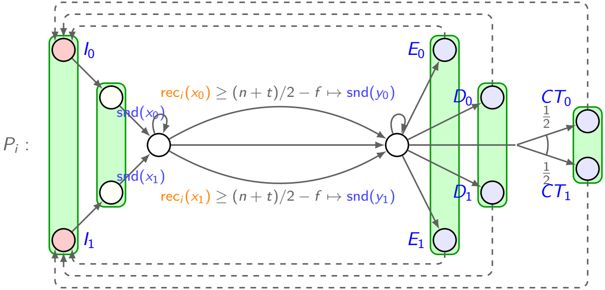


Need for refined model for probabilistic threshold automaton with message IDs, process IDs, etc.

Probabilistic threshold automata with IDs

Illustration on Ben Or's algorithm



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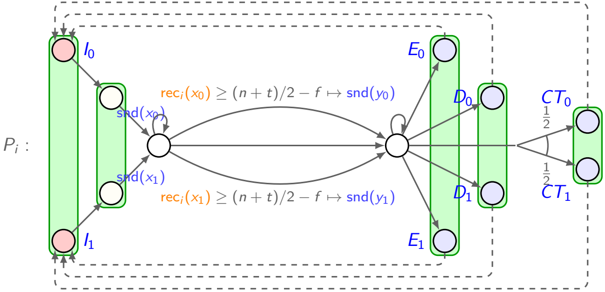

- local variables count received messages of each type
- global set of sent messages
- equivalence on locations that **weak** adversaries do not distinguish

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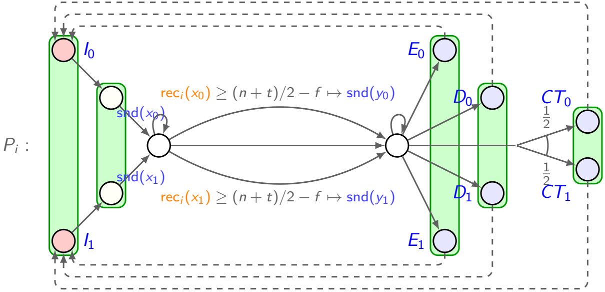

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Specifications still in $\mathsf{ELTL}_{\mathsf{FT}}$
atomic propositions: whether some/no process is in ℓ at round k

Reduction to round-rigid adversaries

For every **weak** adversary a, there is a **round-rigid weak** adversary a′ such that for every specification φ in $\mathsf{ELTL_{FT}}$, $\mathbb{P}_{\mathsf{a}}(\varphi) = \mathbb{P}_{\mathsf{a'}}(\varphi)$.

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- transform into a communication-closed adversary
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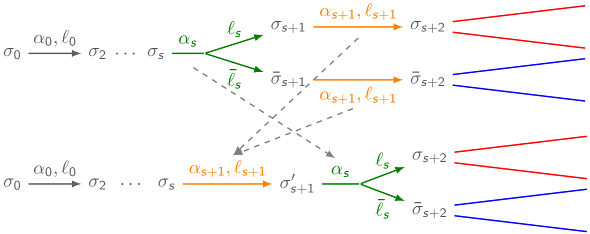
- further transform into a round-rigid adversary by re-ordering swapped transitions



Reduction to round-rigid adversaries

For every **weak** adversary a, there is a **round-rigid weak** adversary a$'$ such that for every specification φ in $\mathsf{ELTL_{FT}}$, $\mathbb{P}_{\mathsf{a}}(\varphi) = \mathbb{P}_{\mathsf{a'}}(\varphi)$.

Key ideas

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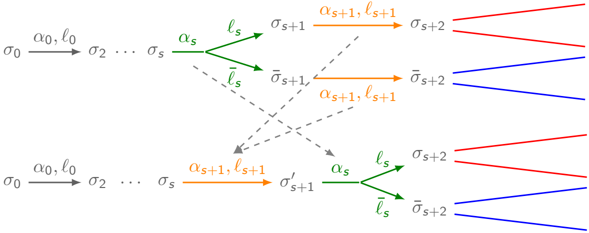
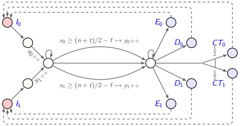


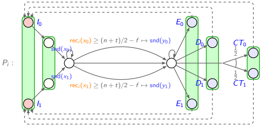
establish correspondence between models with and without IDs

Outline

Contributions

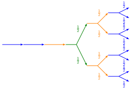
- Modeling of randomized fault-tolerant distributed algorithms



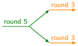


probabilistic threshold automata (PTA) probabilistic threshold automata with IDs (PTA-ID)

- Efficient verification techniques for PTA to prove
    - non-probabilistic specs
    - prob. specs under **round-rigid** adversaries



- Experimental validation on randomized consensus algorithms
- Verification framework for PTA-ID to prove
    - non-prob. and prob. specs under **weak** adversaries



On-going and future work

- Formalisation of correspondence between PTA and PTA-ID

- Structural conditions to enable reordering for strong(er) adversaries
- Quantitative verification techniques for performance evaluation

  average number of rounds before termination

- Models and verification techniques for other classes of randomized distributed algorithms

  global coin tosses

  randomized adversary

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Nirringrazzjak