Verification of population protocols with unordered data

Steffen van Bergerem
Roland Guttenberg
Sandra Kiefer
Corto Mascle (thanks Corto for the slides!)
Nicolas Waldburger
Chana Weil-Kennedy

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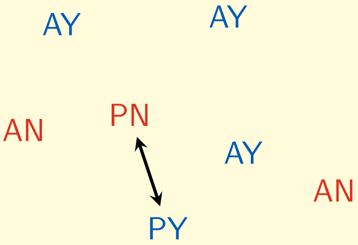

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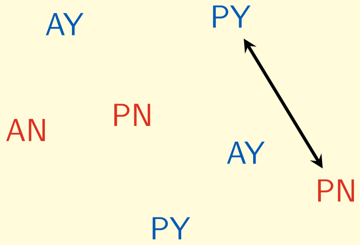

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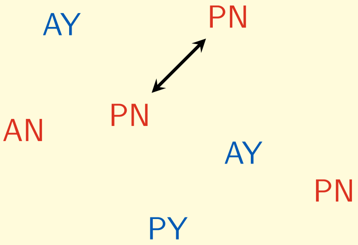

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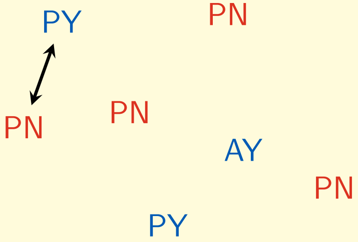

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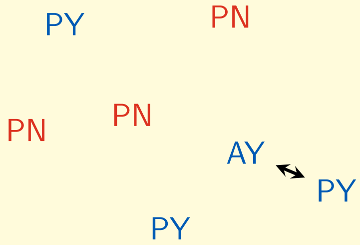

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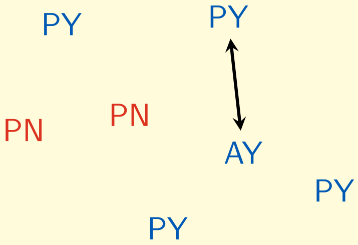

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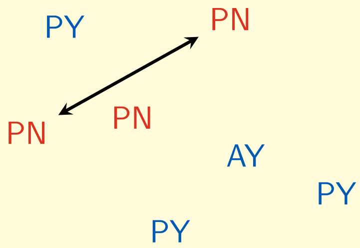

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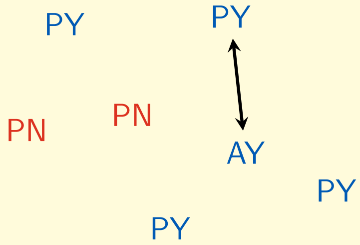

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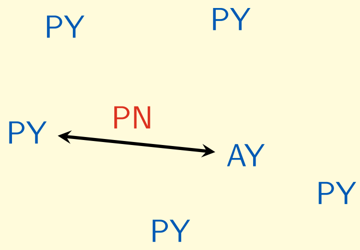

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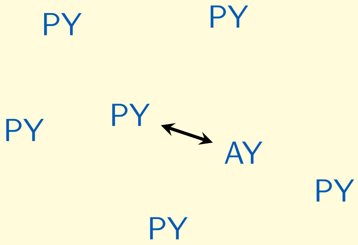

Population Protocols [Angluin, Aspnes, Diamadi, Fischer, Peralta, PODS 2004]

Finite set of states Q, with set $I \subseteq Q$ of *initial states*.
States are partitioned in two opinions $Q = Q_{Yes} \sqcup Q_{No}$
Interactions $\Delta \subseteq Q^2 \times Q^2$.

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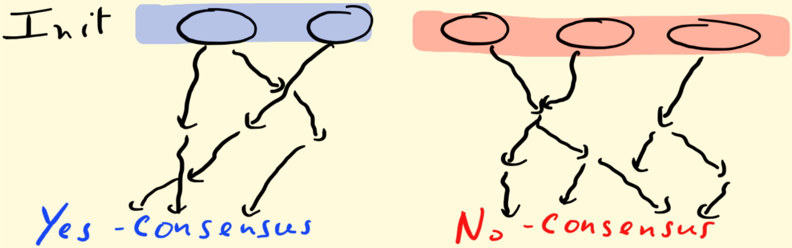
- Random pairwise interactions
- Stable consensus is reached when everyone agrees on Yes or No and no one can ever change their mind

A protocol is **well-specified** if from all initial configuration, either a Yes-consensus is reach with proba 1, or a No-consensus is reached with proba 1.



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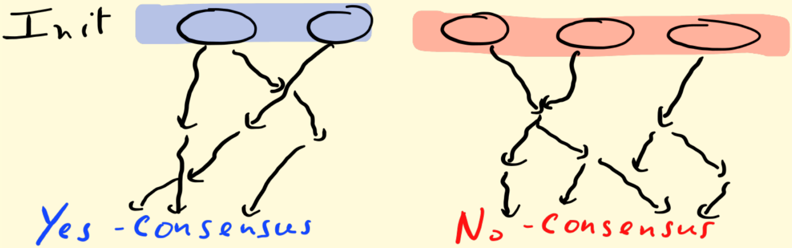

The **predicate** computed by the protocol is then the set of initial configurations from which we reach a Yes-consensus.

First questions

Which predicates can be computed by population protocols?

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Can we check if a population protocol is well-specified? ✔

Theorem [Esparza, Ganty, Leroux, Majumdar 2015]

Checking if a population protocol is well-specified is **decidable** but as hard as Petri net reachability (Ackermann-complete).

Population Protocols with Unordered Data

Defined by Michael Blondin and François Ladouceur [ICALP'23]
Each agent carries a permanent datum taken from an infinite set $\mathbb{D}$.
Interactions: $\Delta \subseteq Q^2 \times \{=, \neq\} \to Q^2$
Interactions take into account whether the two agents have $=$ or $\neq$ data.

$$\left.\begin{matrix} q_0, x \\ \\ q_2, y \end{matrix}\right\} \xrightarrow[x \neq y]{} \left\{\begin{matrix} q_1, x \\ \\ q_3, y \end{matrix}\right.$$

Majority predicate

Does some datum have more agents than all other combined?

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Open problem

What are the predicates computed by PPUD?

Well-specification problem

Given a PPUD, is it well-specified?

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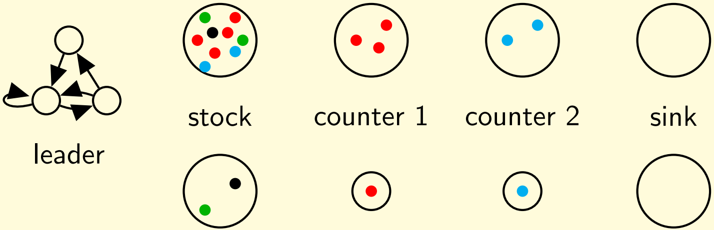
Theorem [Us, ICALP'24]

It is **undecidable** to check whether a PPUD is well-specified.

- Simulate a 2-counter machine with zero-tests.



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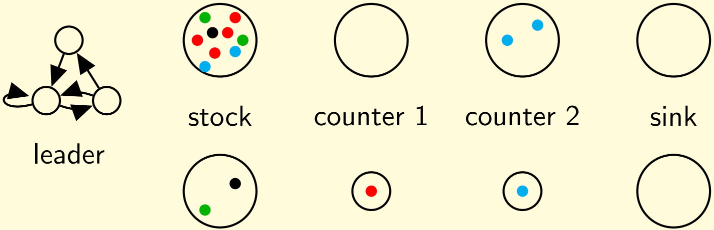

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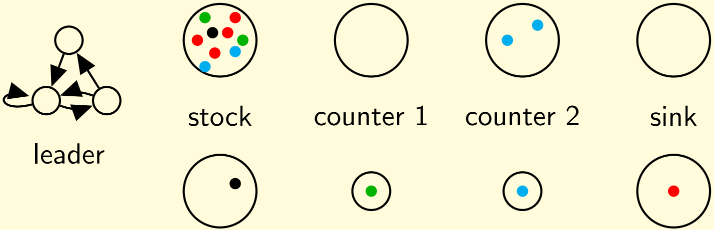

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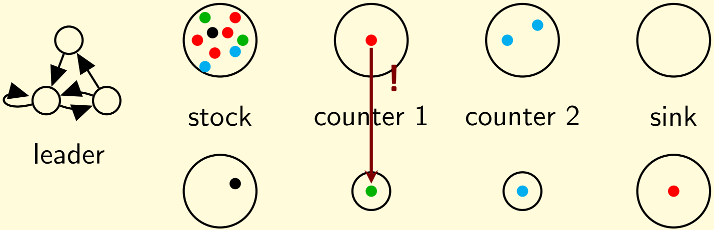

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Theorem (Esparza, Ganty, Majumdar, Weil-Kennedy 2018)

Well-specification is PSPACE-complete for Immediate-Observation population protocols **without data**.

Interval predicate = Boolean combination of
"At least 3 distinct data with between 1 and 3 agents in state q and 4 agents in state q'".

$$\exists d_1, d_2, d_3, \bigwedge_{i=1}^{3} (1 \leq \#(q, d_i) \leq 3) \wedge (4 \leq \#(q, d_i))$$

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Theorem [Blondin, Ladouceur 2023]

The predicates computed by IOPPUD are exactly interval predicates.

IOPPUD

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Key lemma

Given a set of configurations C described by an interval predicate, we can compute interval predicates expressing $Pre^*(C)$ and $Post^*(C)$.

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Copycat: in an IOPPUD, if an agent with datum d goes from q_1 to q_2 then we can send as many agents with datum d as we want from q_1 to q_2: the observed agent is still here.

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Copycat: in an IOPPUD, if an agent with datum d goes from q_1 to q_2 then we can send as many agents with datum d as we want from q_1 to q_2: the observed agent is still here.
Using this fact, we prove that we can rearrange any run so that

- each datum only has a limited number of agents that get observed during the run.
- only a limited number of data have agents that are observed by other data.

IOPPUD

Generalised Reachability Expressions:

$$E ::= \text{Interval Predicate} \mid E \cup E \mid \overline{E} \mid \mathit{Pre}^*(E) \mid \mathit{Post}^*(E)$$

Question: given a GRE E, do we have $[\![E]\!]_{\mathcal{P}}$?

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Example: The protocol is well-specified if and only if

$$\Gamma_0 \cap \mathit{Pre}^*(\overline{\mathit{Pre}^*(\text{Stable}_{\mathit{Yes}})}) \cap \mathit{Pre}^*(\overline{\mathit{Pre}^*(\text{Stable}_{\mathit{No}})}) = \emptyset$$

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Theorem

Given a GRE E, we can compute an interval predicate for $[\![E]\!]_{\mathcal{P}}$.

Corollary

Given a GRE E, we can check if $[\![E]\!]_{\mathcal{P}} = \emptyset$.

Decidable problems on IOPPUDs

Many problems can be reduced to the emptiness of a GRE:

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Many problems can be reduced to the emptiness of a GRE:

- `Well-specification`
  = The protocol computes something
- `Correctness`
  = The protocol computes predicate P
- `Visible termination`
  = all consensus are stable consensus
- `Home-space problem`
  = Every fair run eventually reaches set of configurations H

Complexity

Emptiness of Generalised Reachability Expressions is:

In EXPSPACE

$\rightarrow$ By controlling the growth of coefficients when translating GRE to Interval Predicates.

NEXPTIME-hard

$\rightarrow$ By encoding the tiling of an exponential grid.

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Thanks!