How to achieve Reachability in Broadcast Networks?

An ongoing work

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14 octobre 2024, Grenoble

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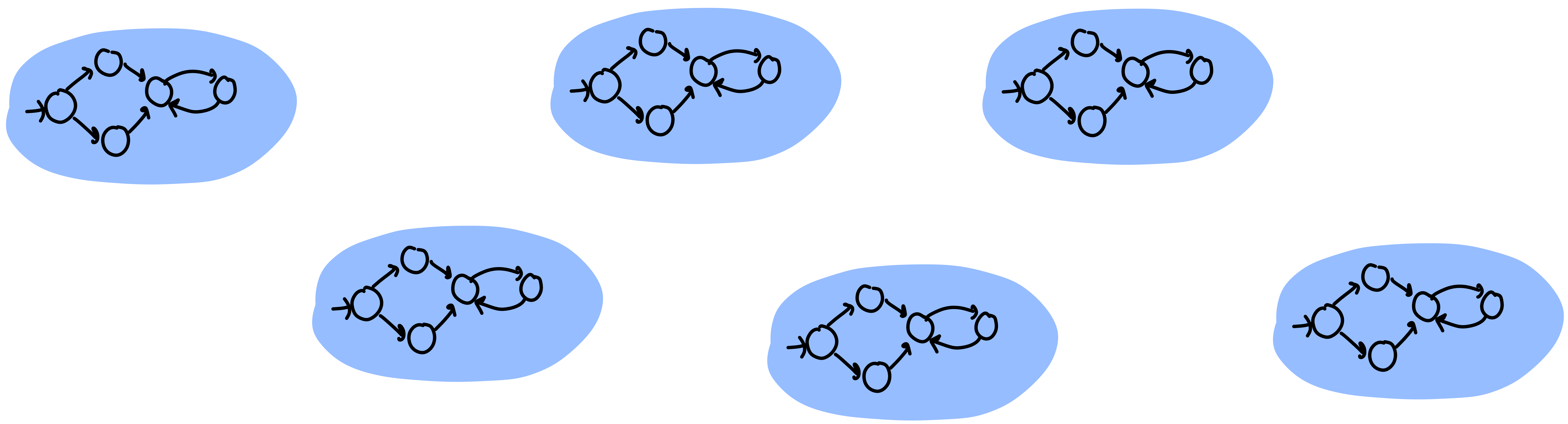
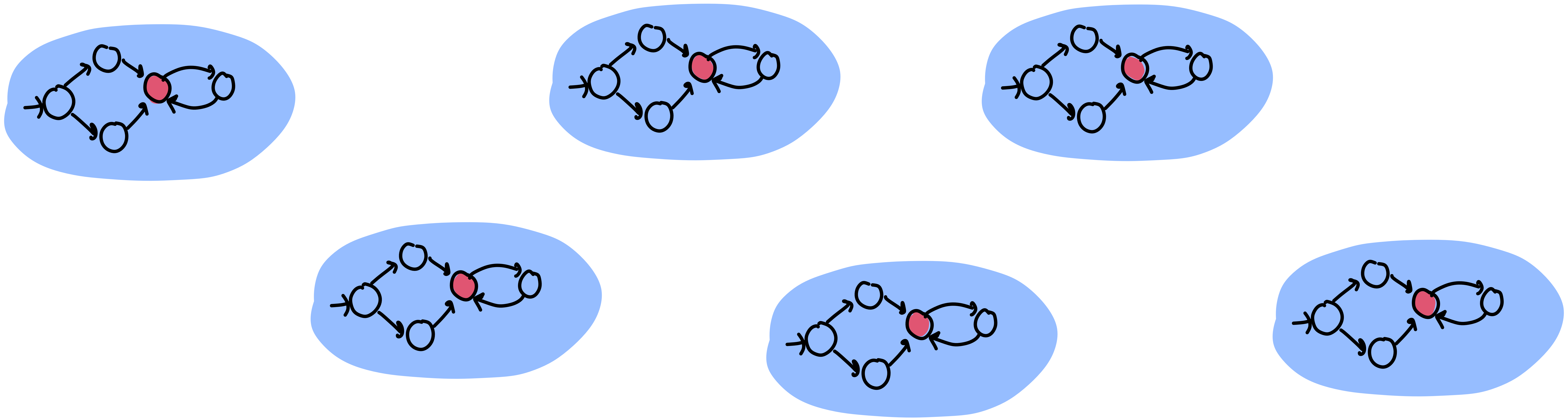
Parameterized Broadcast Networks



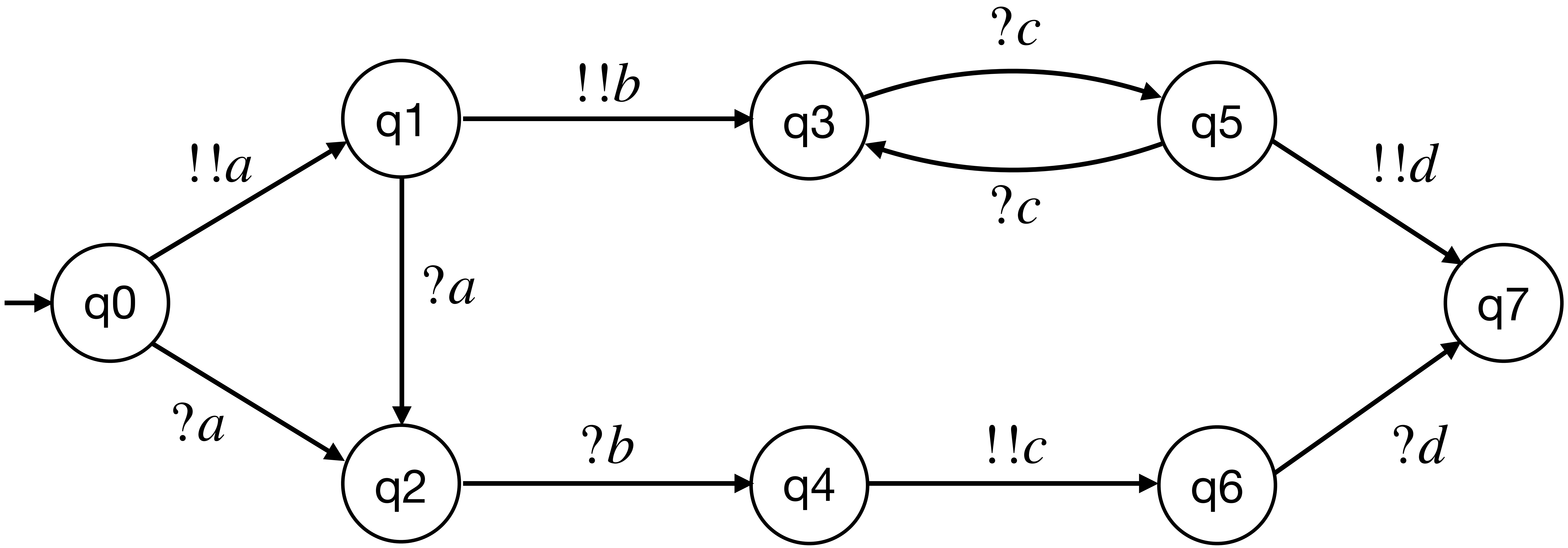
- Unknown number of agents
- Each agent follows a protocol given as a finite-state machine
- Synchronous Communication (Broadcast)
- Interleaving Semantics

The Reachability Question

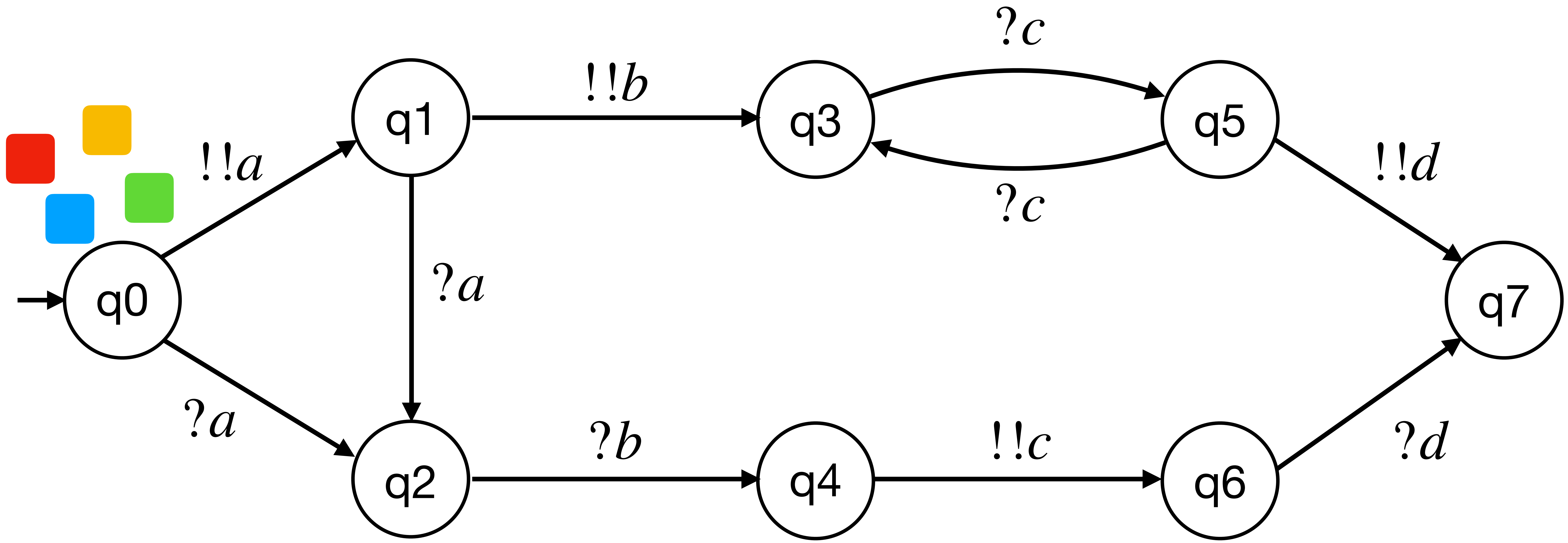


- Is there a number of agents such that there exists a run leading to a bad configuration?

Broadcast Protocols



Example of an execution



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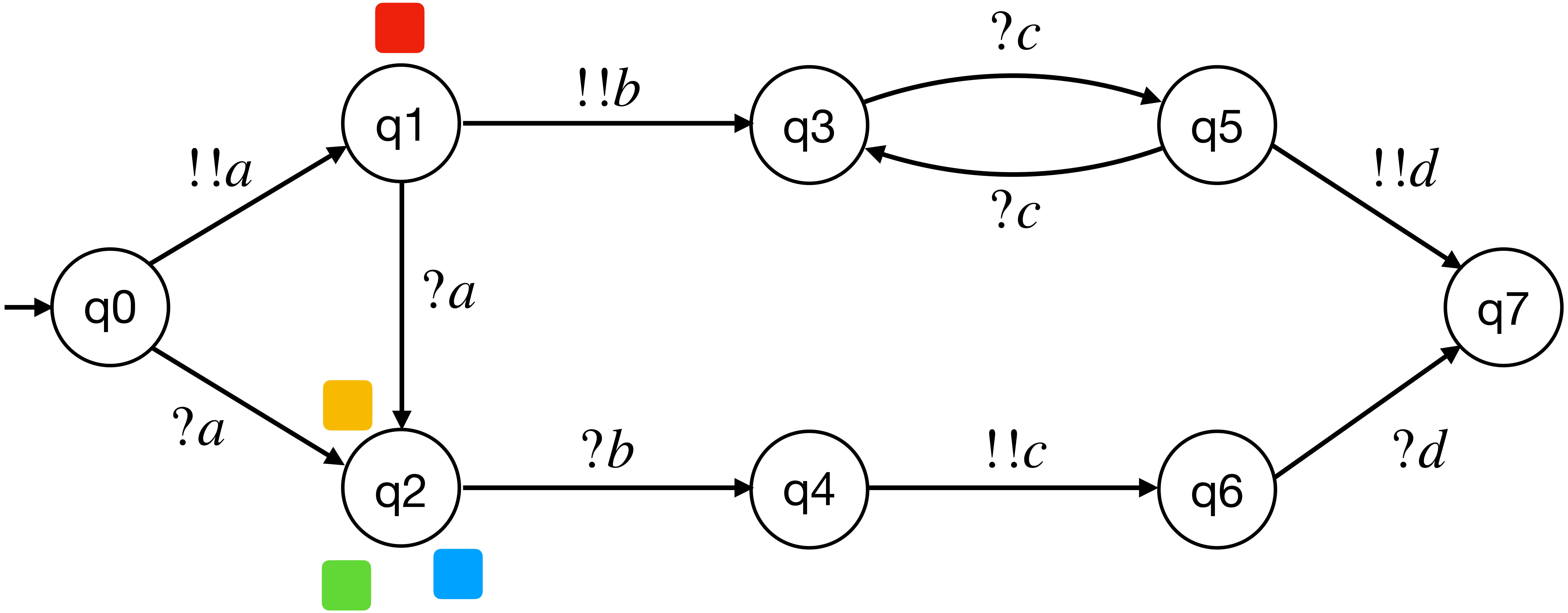

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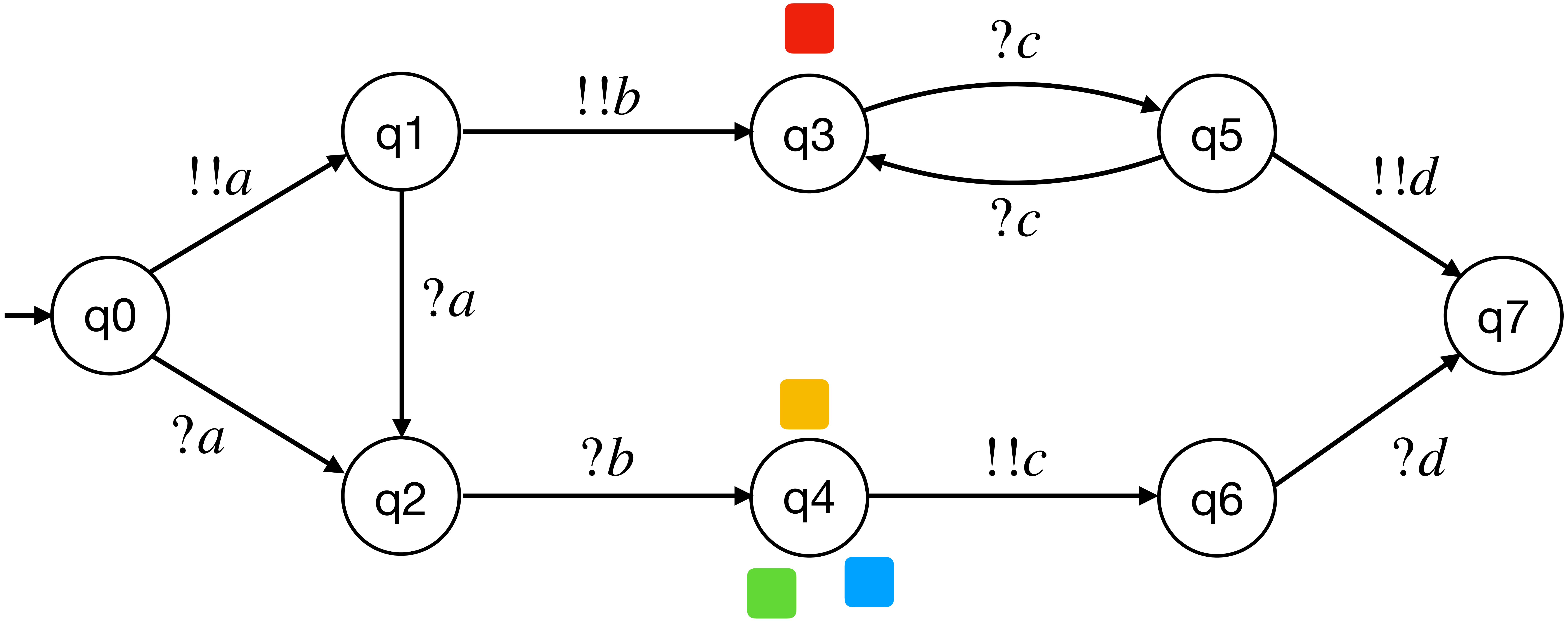

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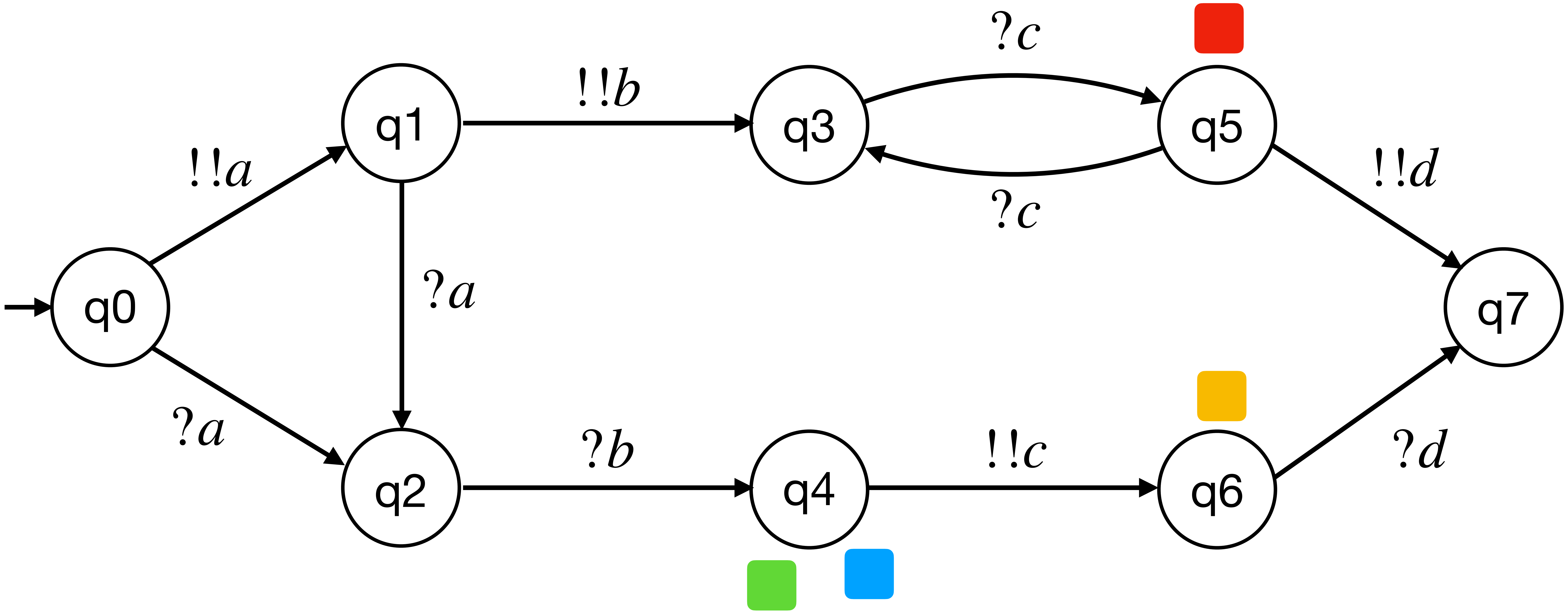

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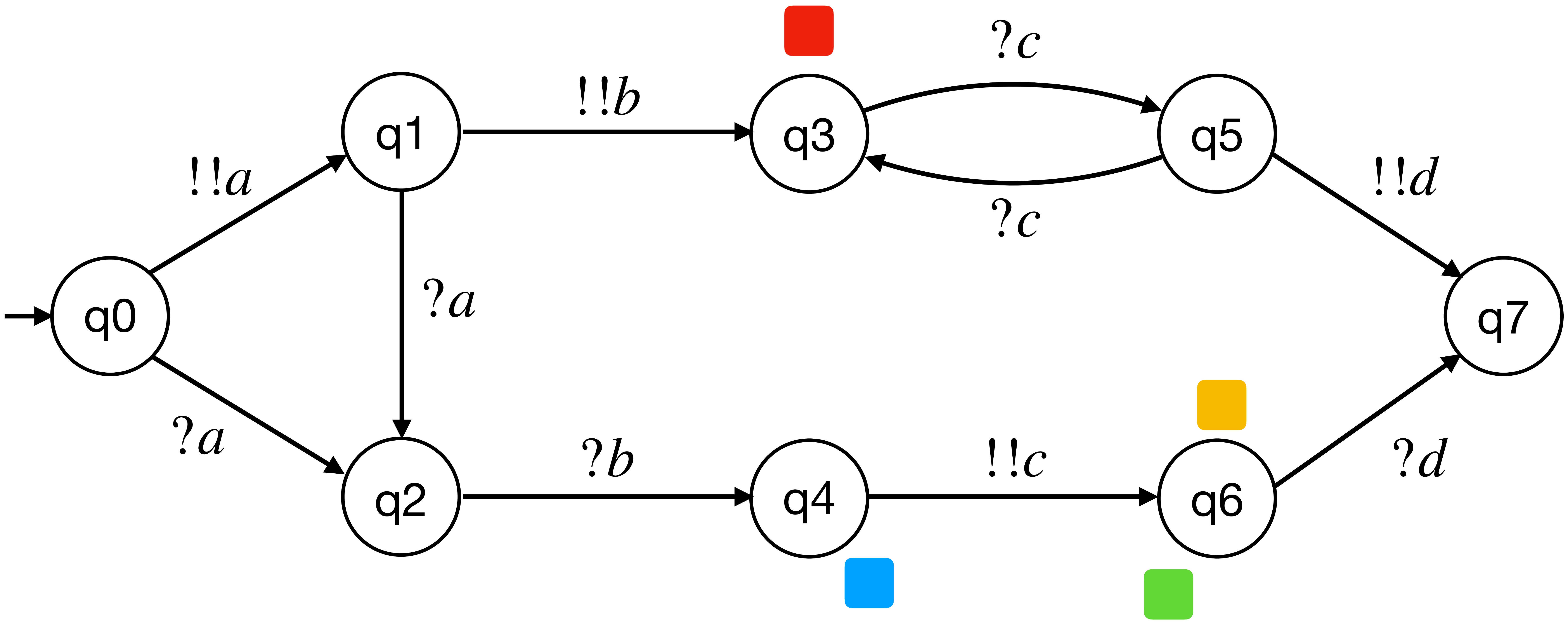

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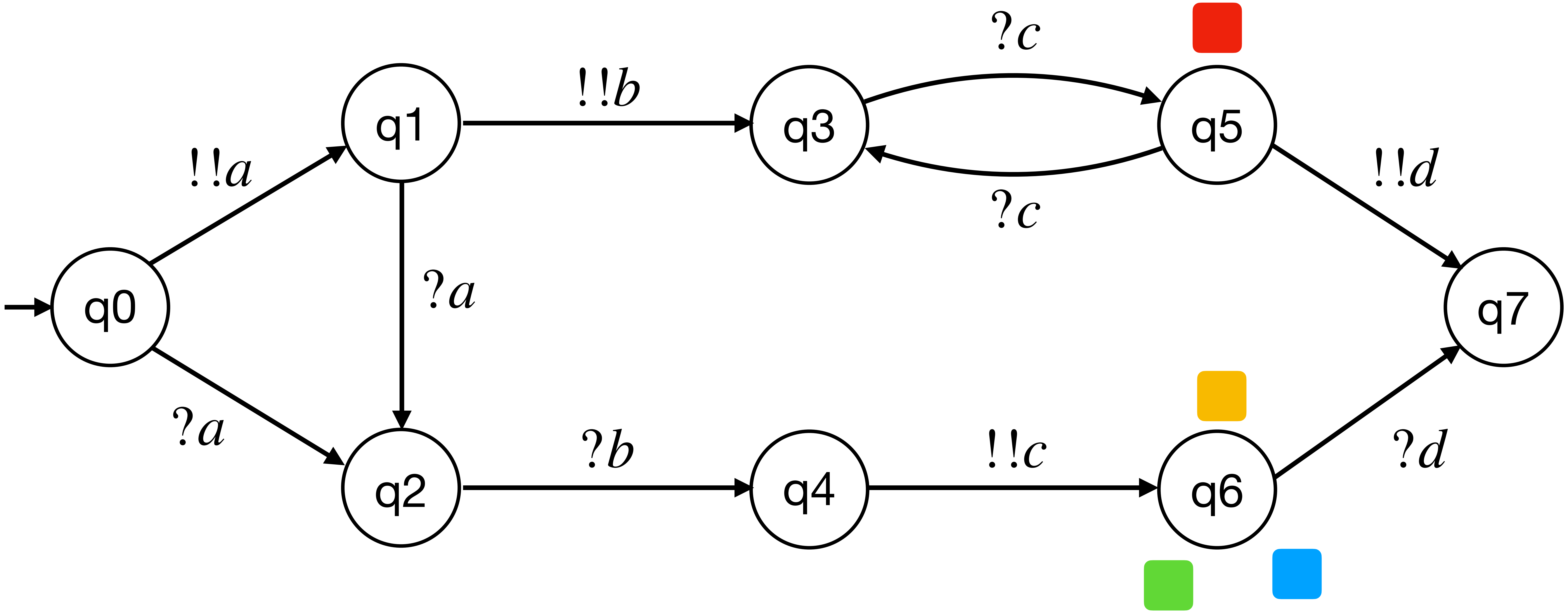

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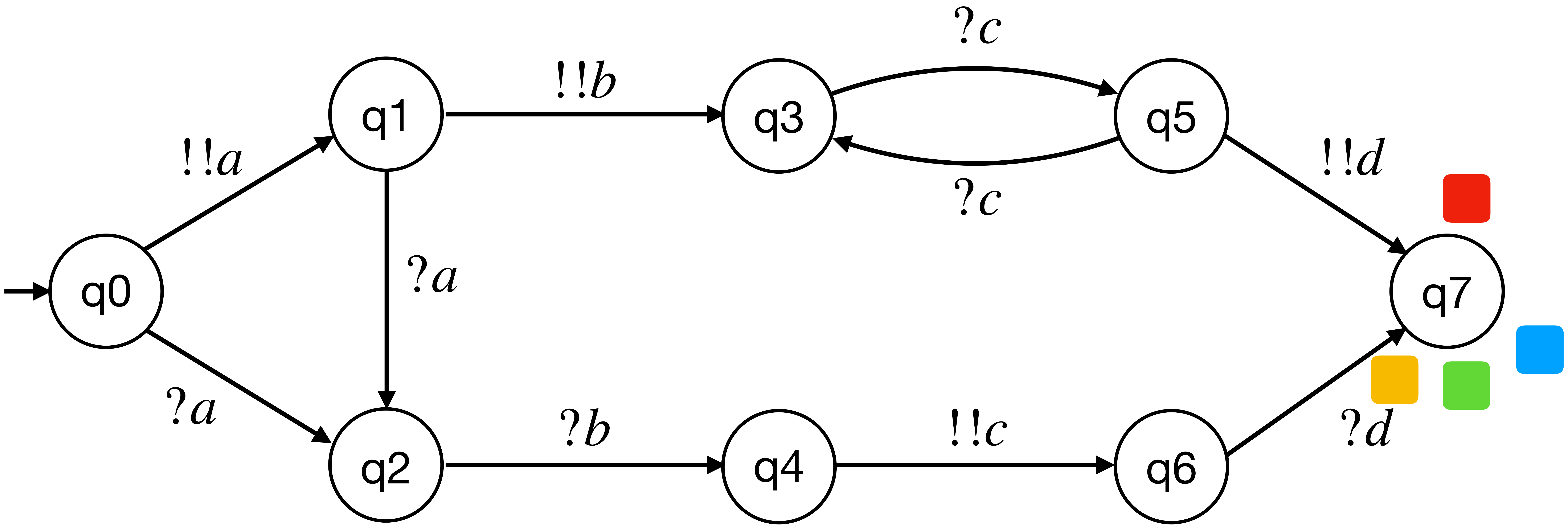
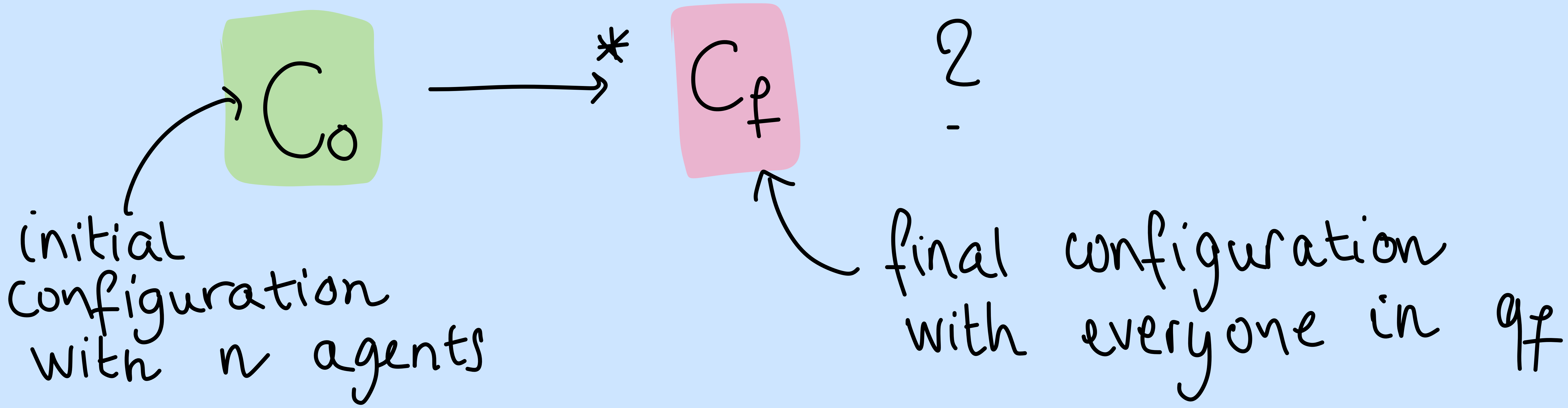


The Reachability Problem Formalized

REACH($\mathscr{P}, q_f$):

Is there a number of agents $n \in \mathbb{N}$ such that:



The Reachability Problem Formalized

REACH($\mathscr{P}, q_f$ **):**

Is there a number of agents $n \in \mathbb{N}$ such that:

initial
configura
with n agents

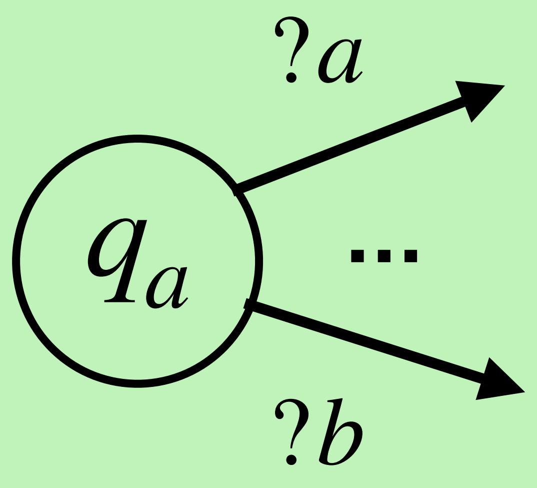
Undecidable

final configuration
with everyone in q_f

A Restriction on Protocols: Wait-Only

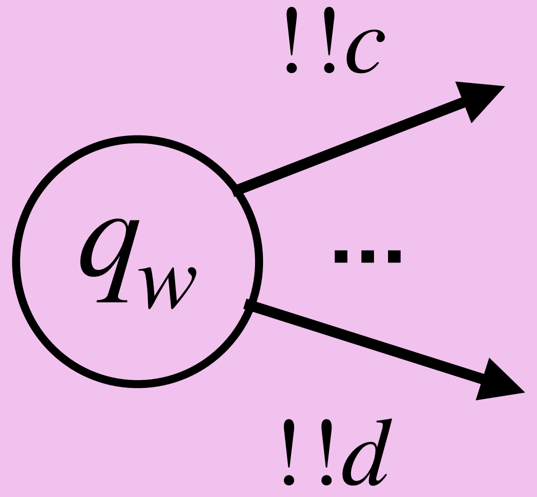
Each state is either:

a waiting state



or

an action state



A Restriction on Protocols: Wait-Only

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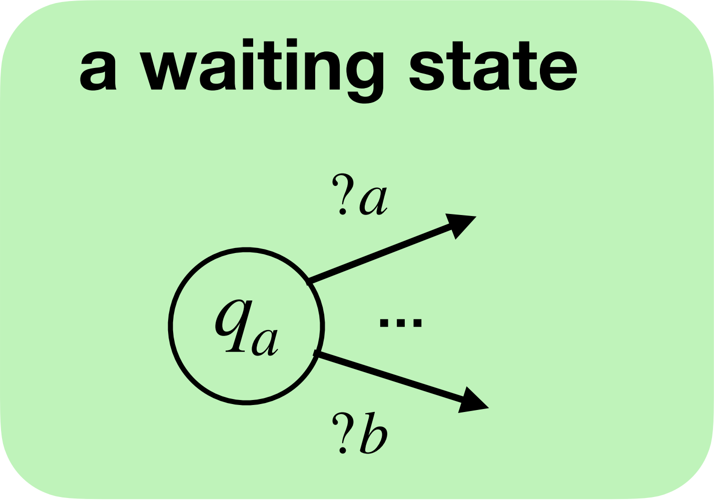
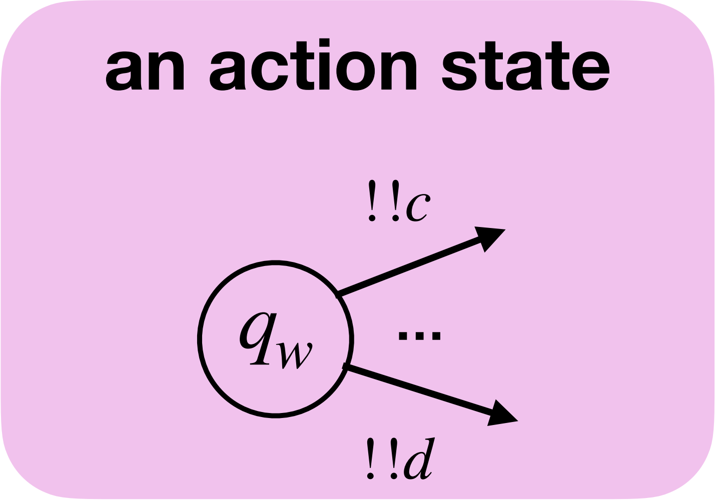
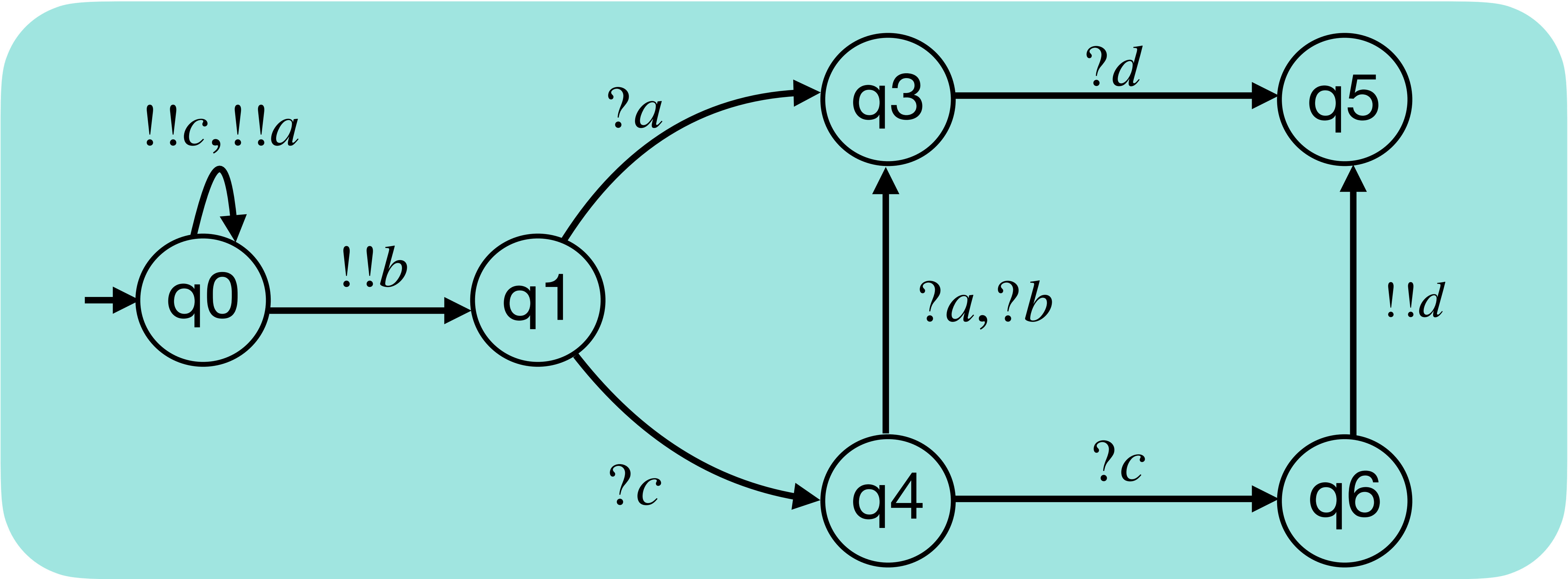


or



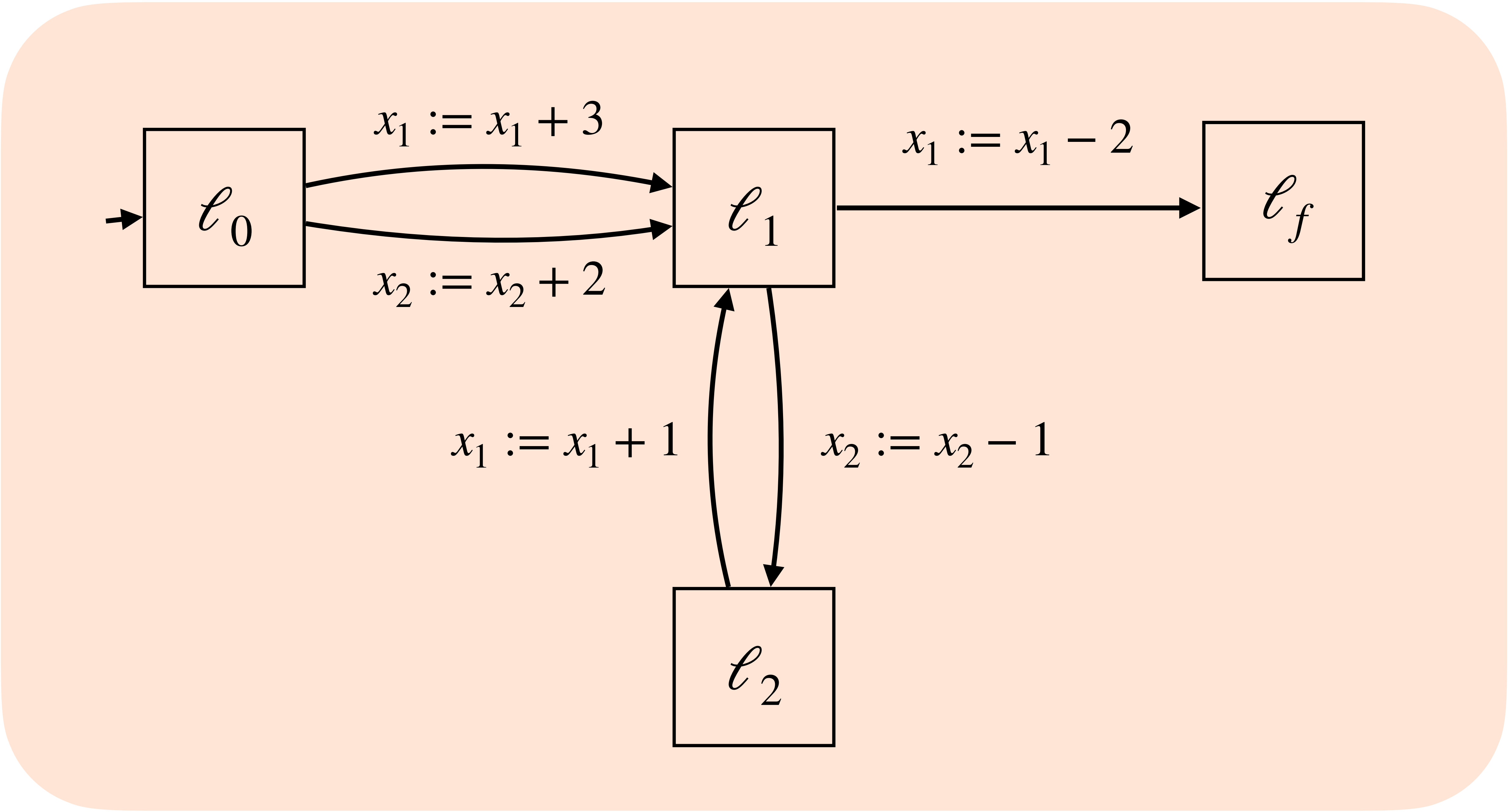


A Wait-Only Protocol

The initial state is always an action state

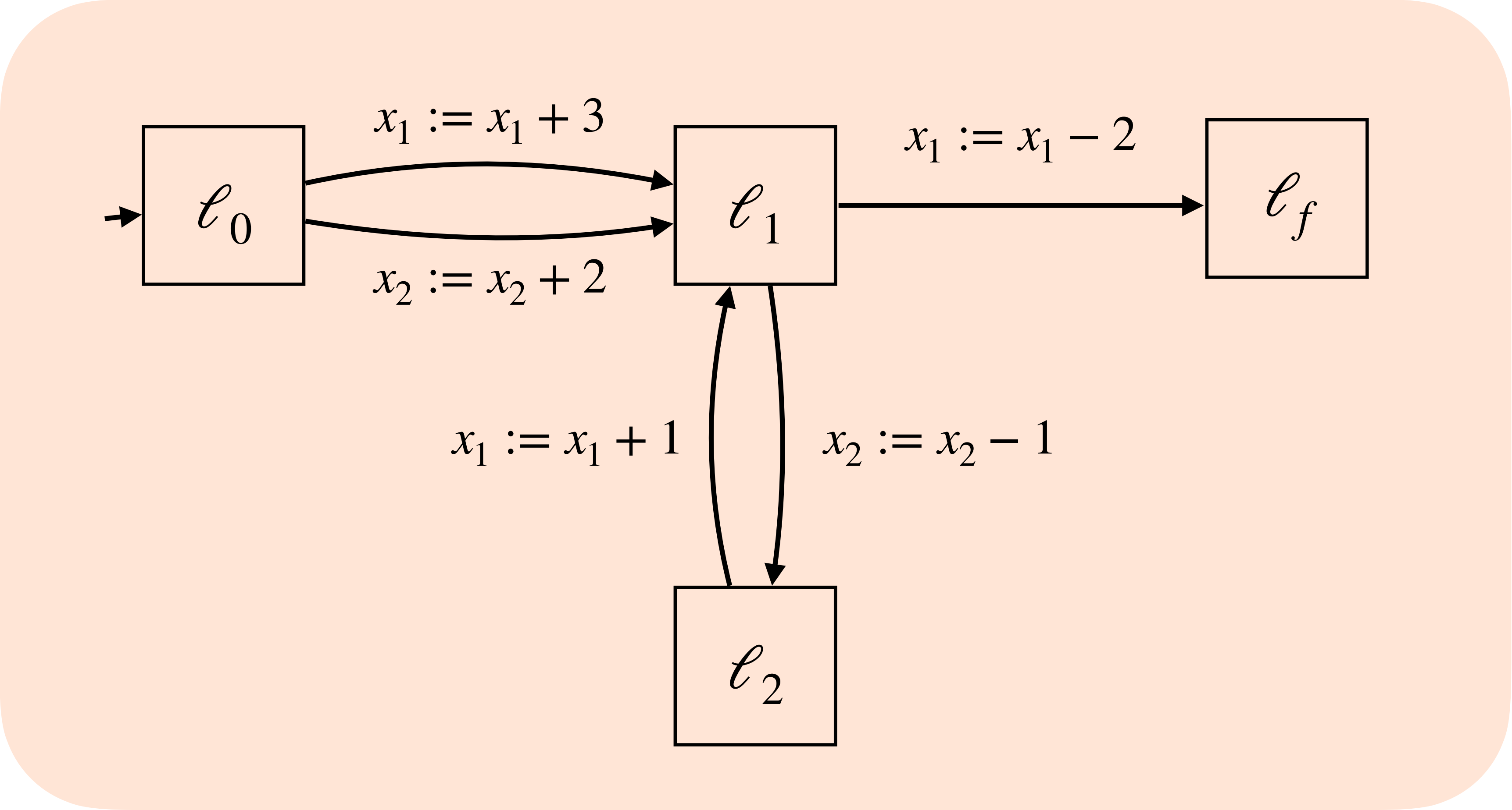
Vector Addition Systems with States

Vector Addition Systems with States



A VASS with two counters x_1, x_2

Vector Addition Systems with States



A VASS with two counters x_1, x_2

	ℓ_0	ℓ_1	ℓ_2	ℓ_1	ℓ_2	ℓ_1	ℓ_f
x_1	0	0	0	1	1	2	0
x_2	0	2	1	1	0	0	0

Reachability in VASS

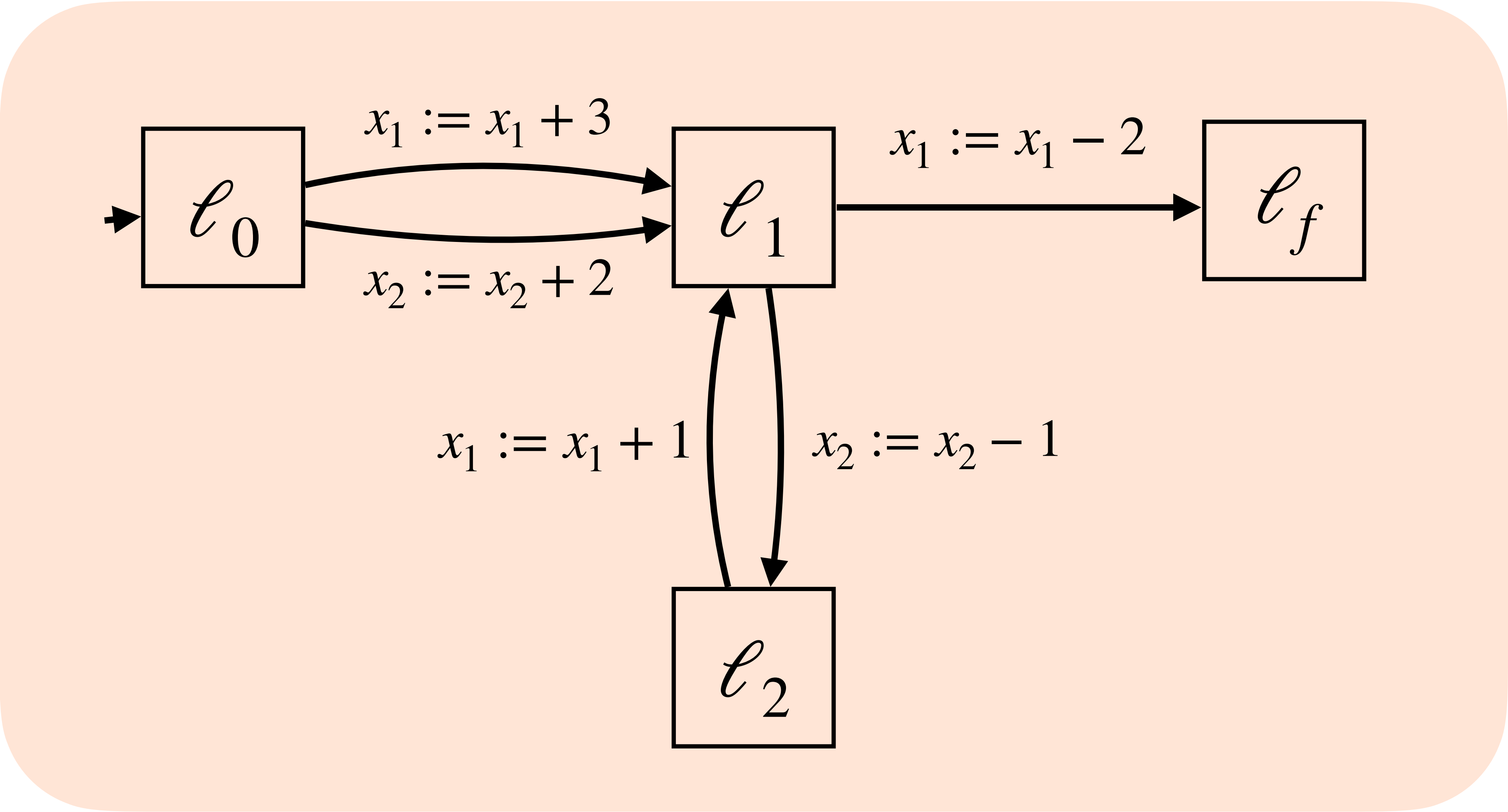
Given a VASS, can we reach $(\ell_f, 0, 0)$ from $(\ell_0, 0, 0)$?

Reachability in VASS

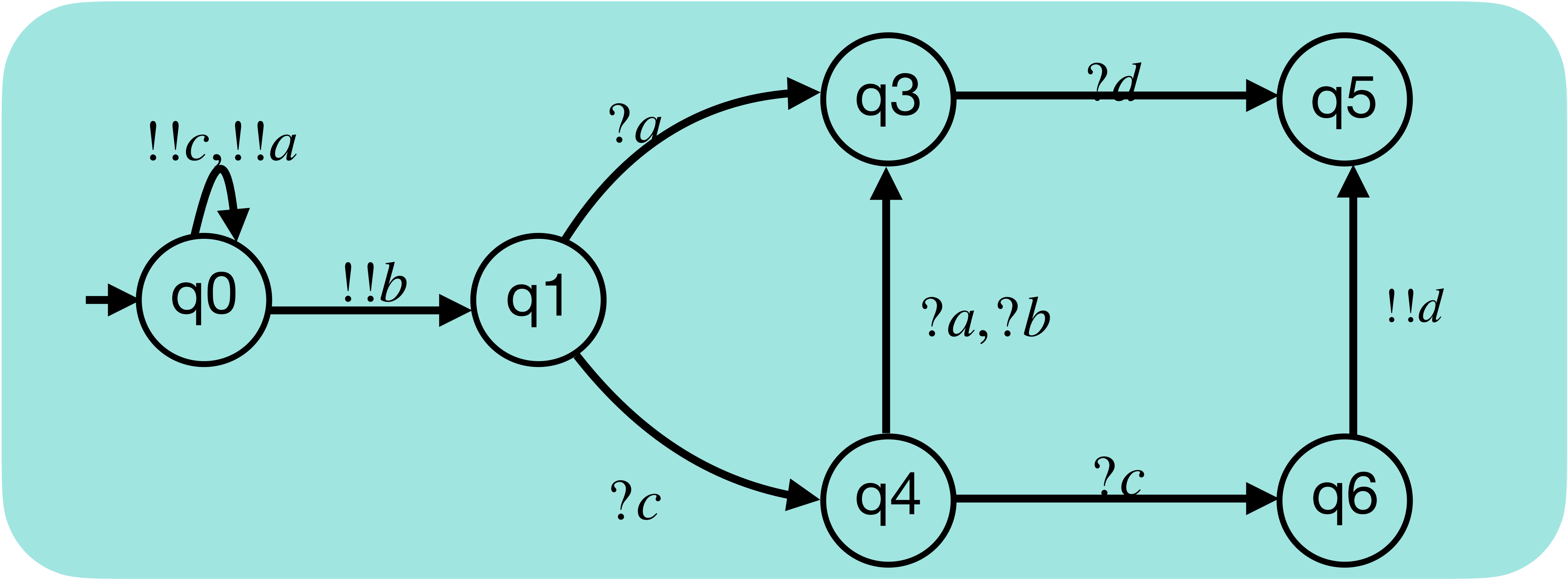
Given a VASS, can we reach $(\ell_f, 0, 0)$ from $(\ell_0, 0, 0)$?

Decidable but Ackermann-hard

[LerouxSchmitz19] [Leroux'21, CwerwinskiOrlikowski'21]

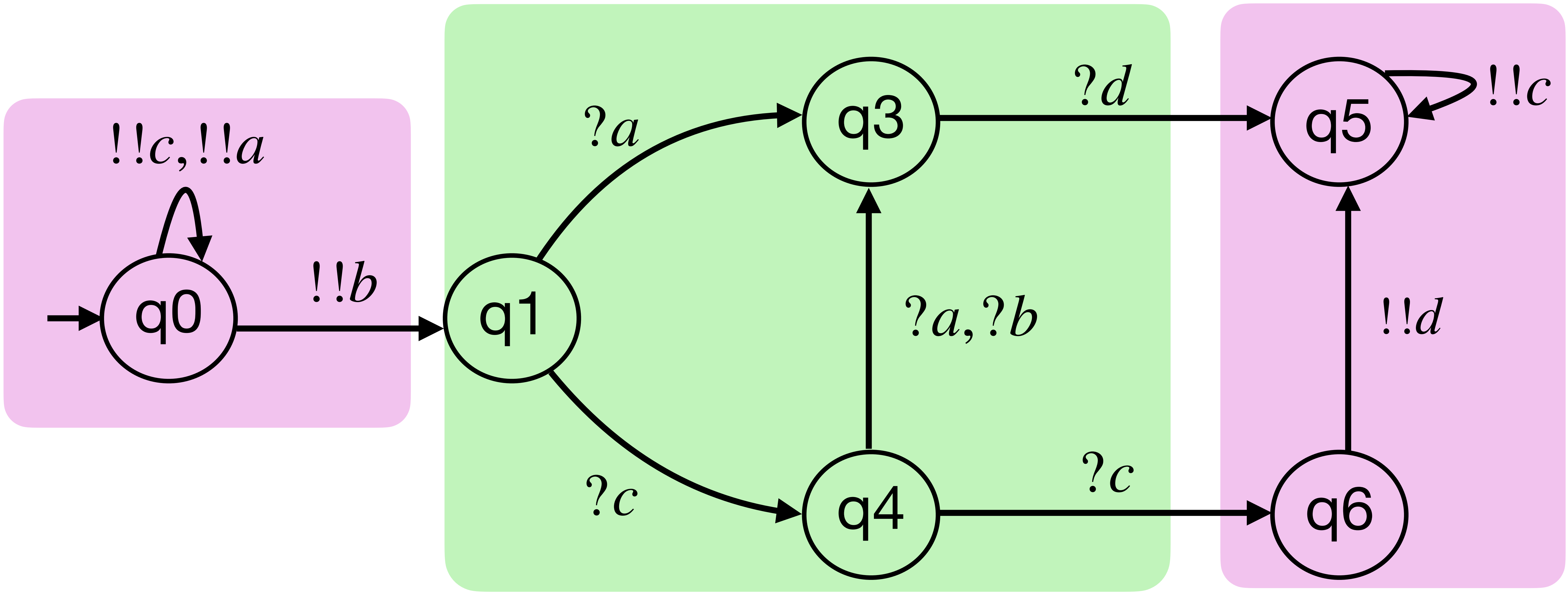


A VASS with two counters x_1, x_2

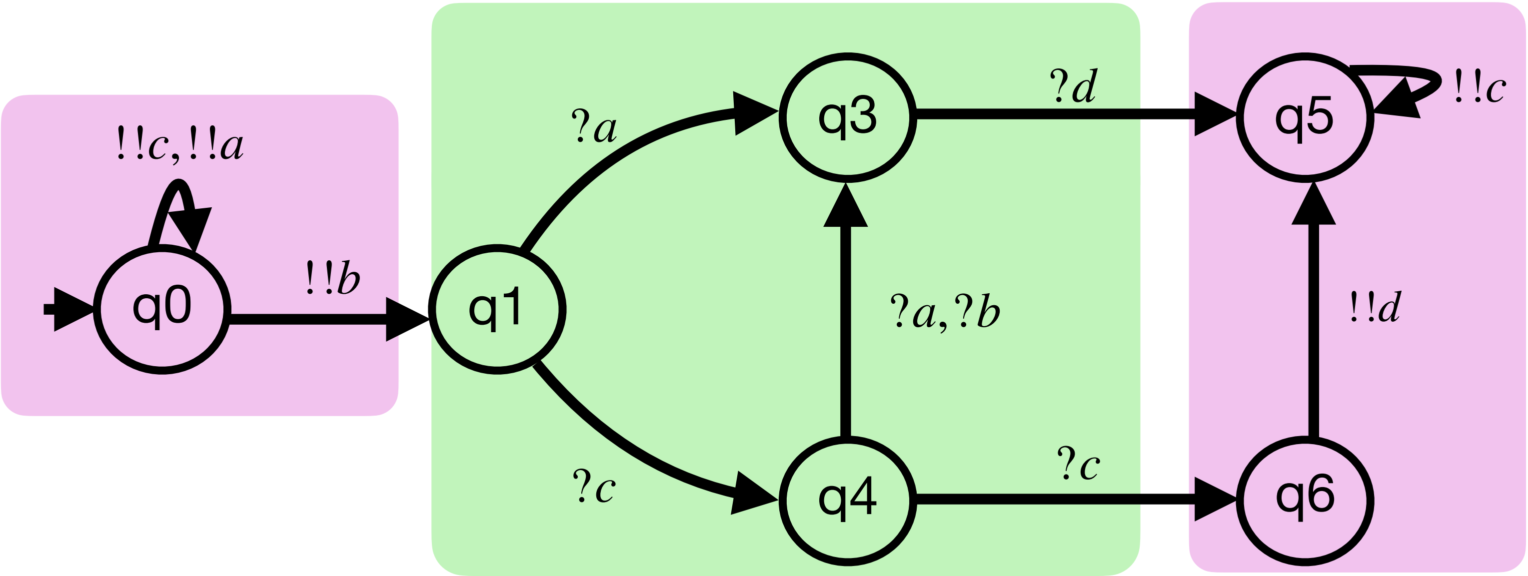


A Wait-Only Protocol

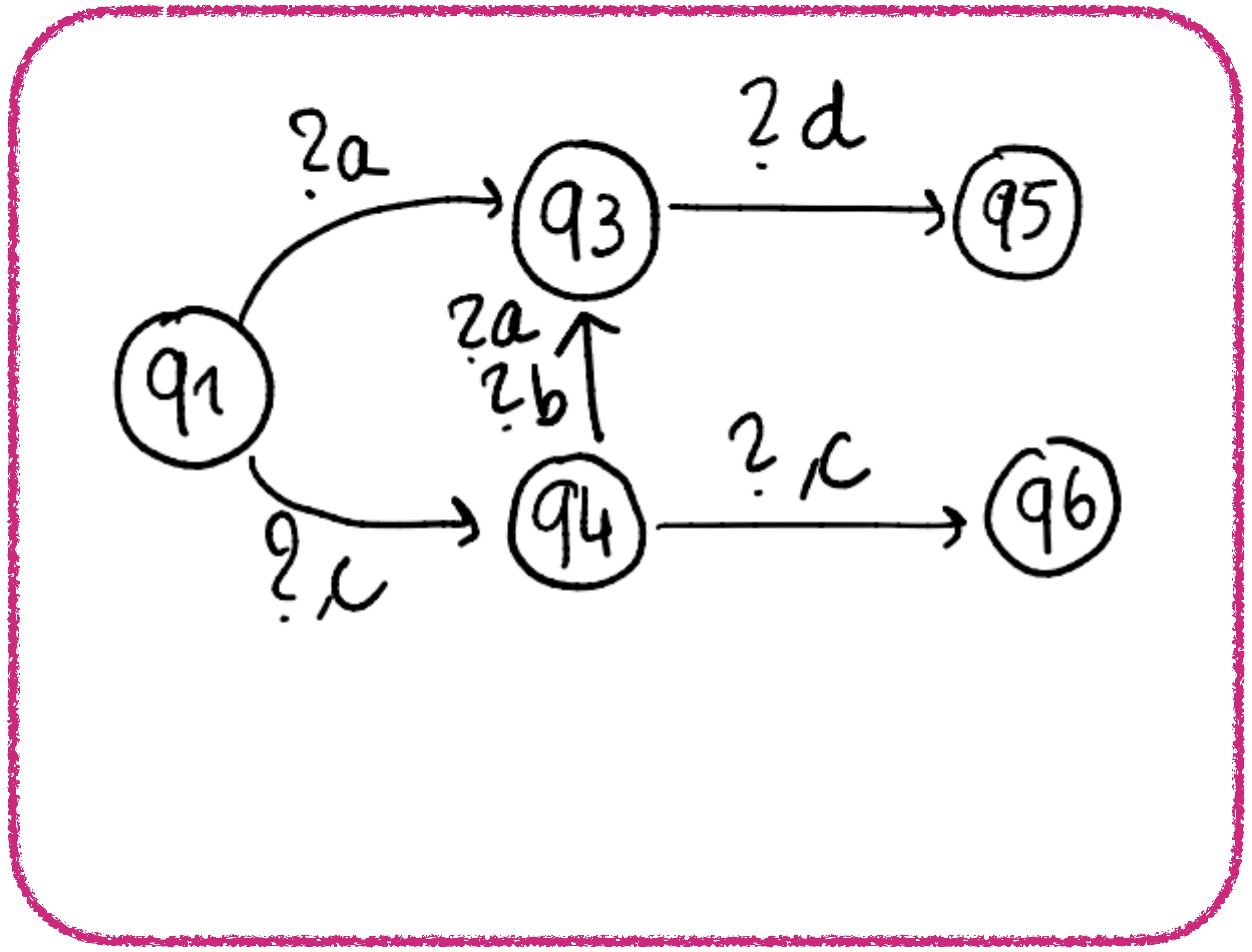
Reductions everywhere!



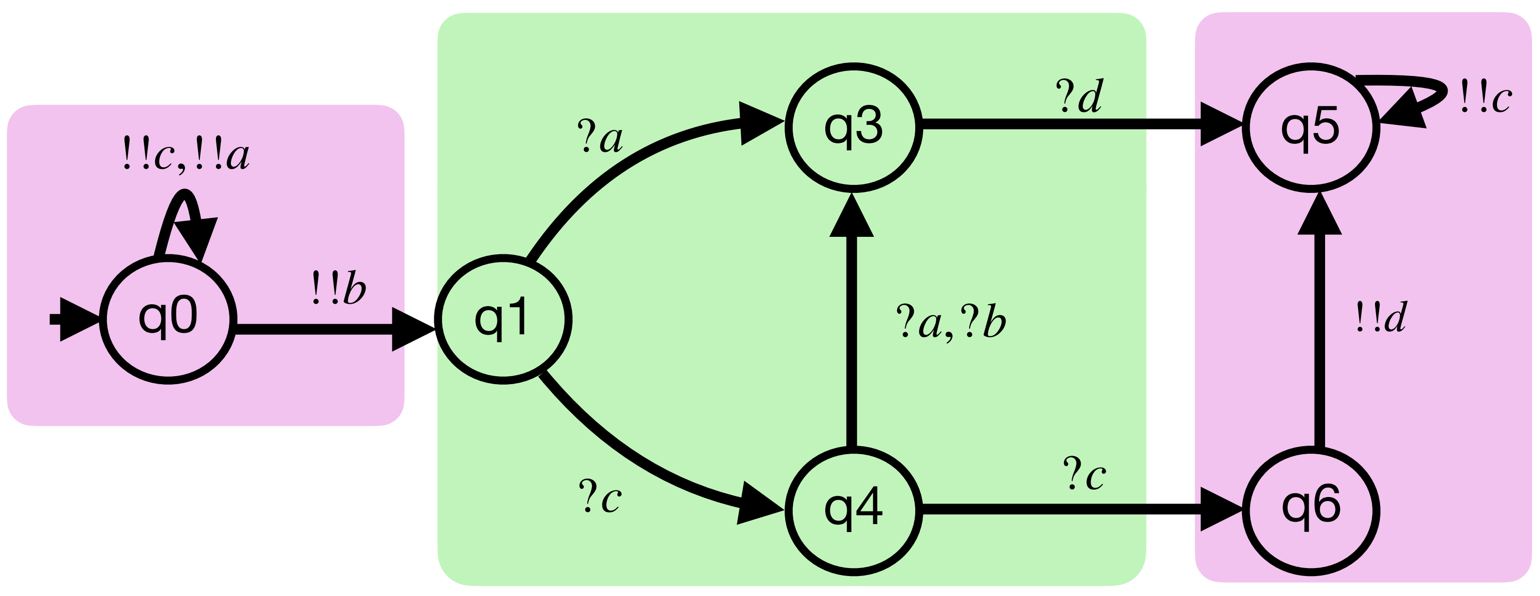
Goal: everyone on q5



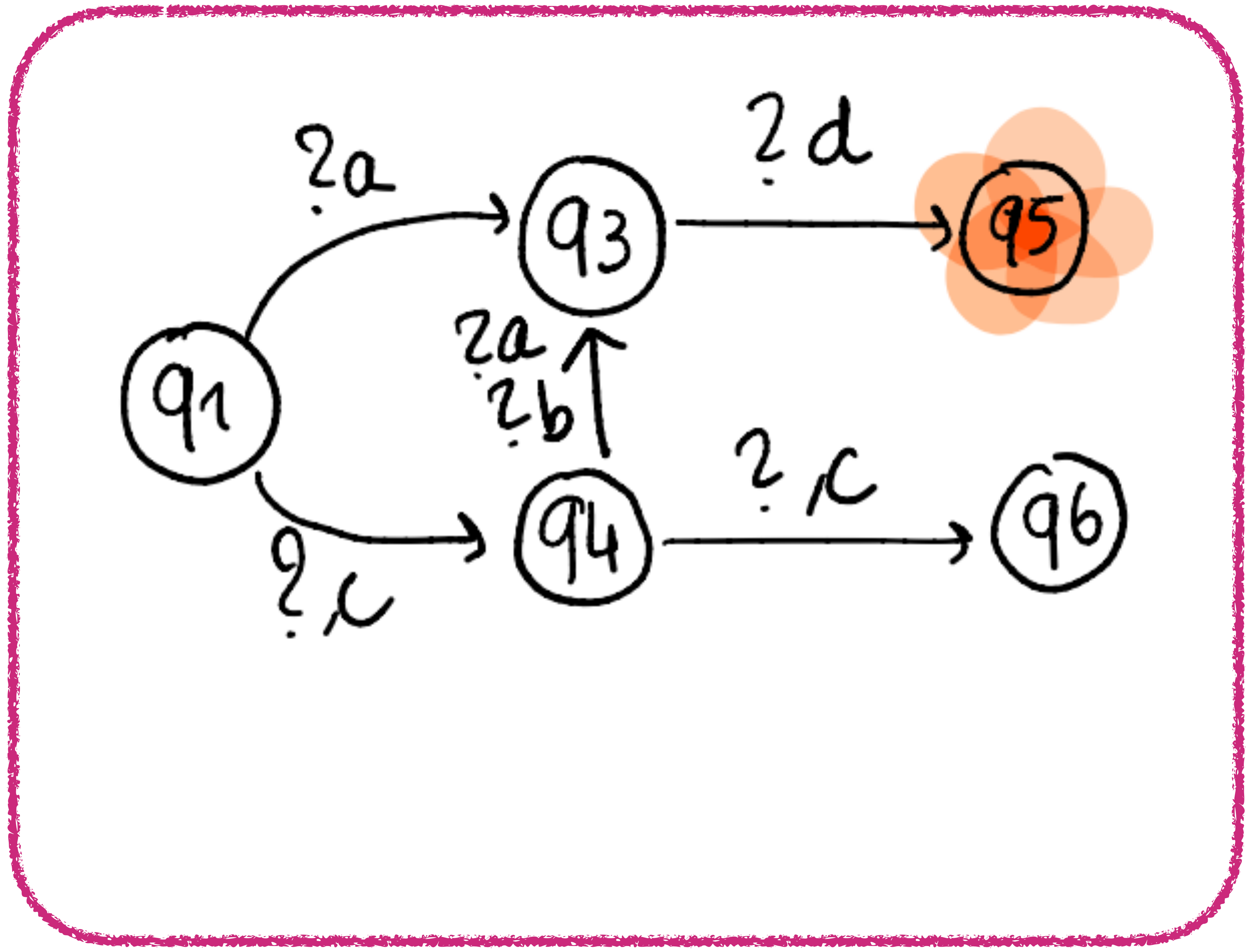
A Summary



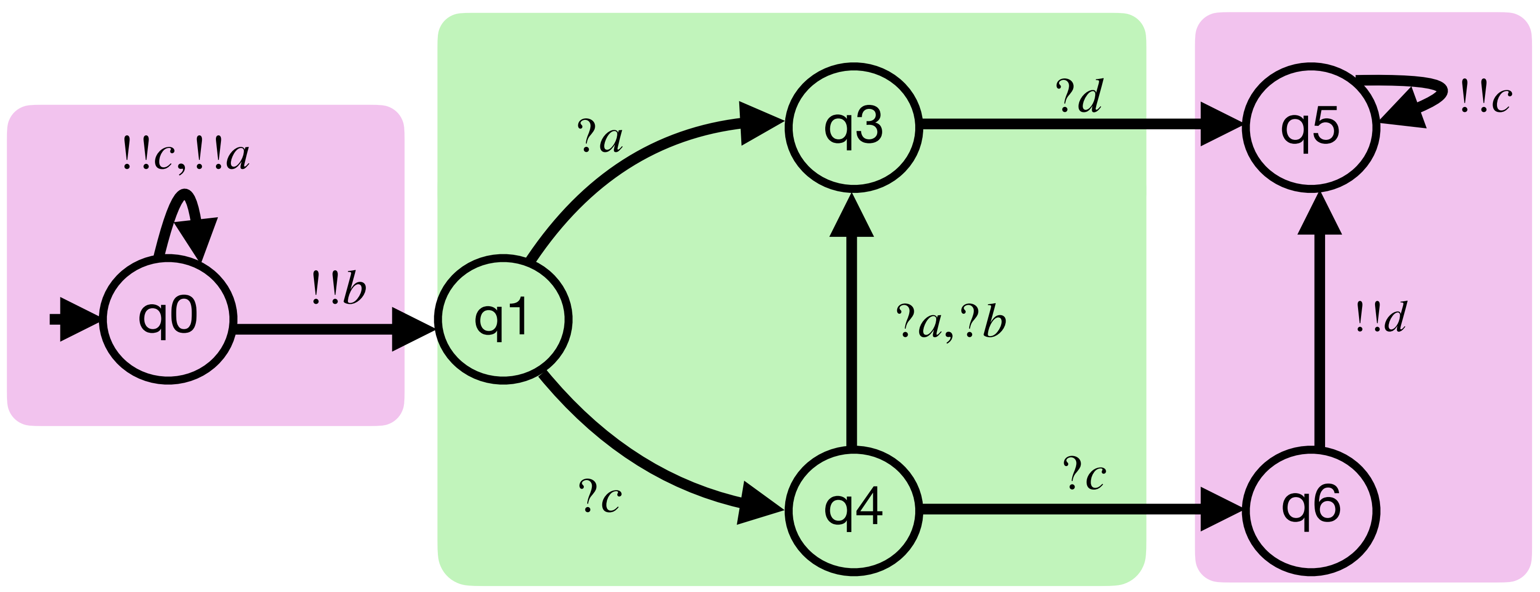
+ one counter



A Summary



+ one counter



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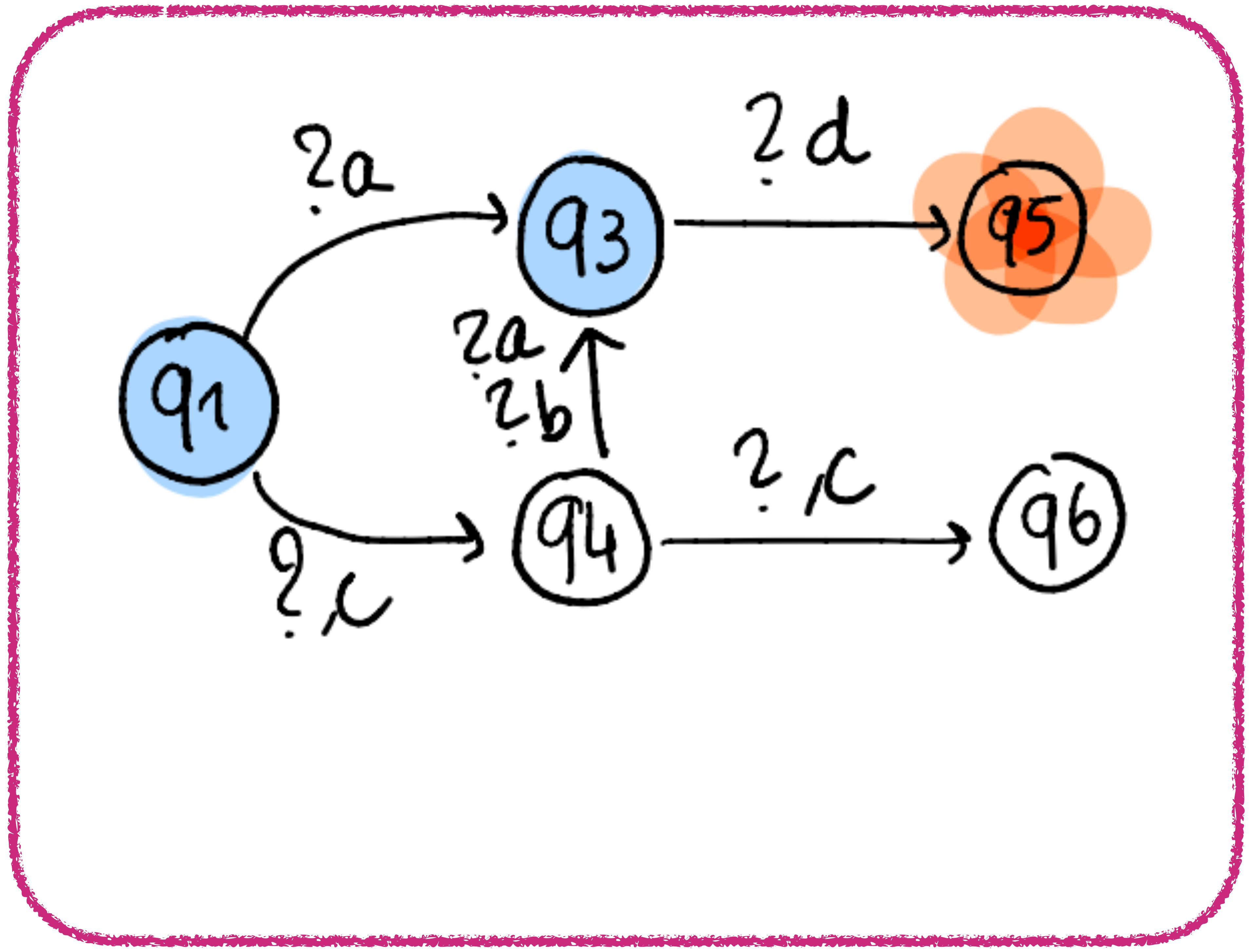
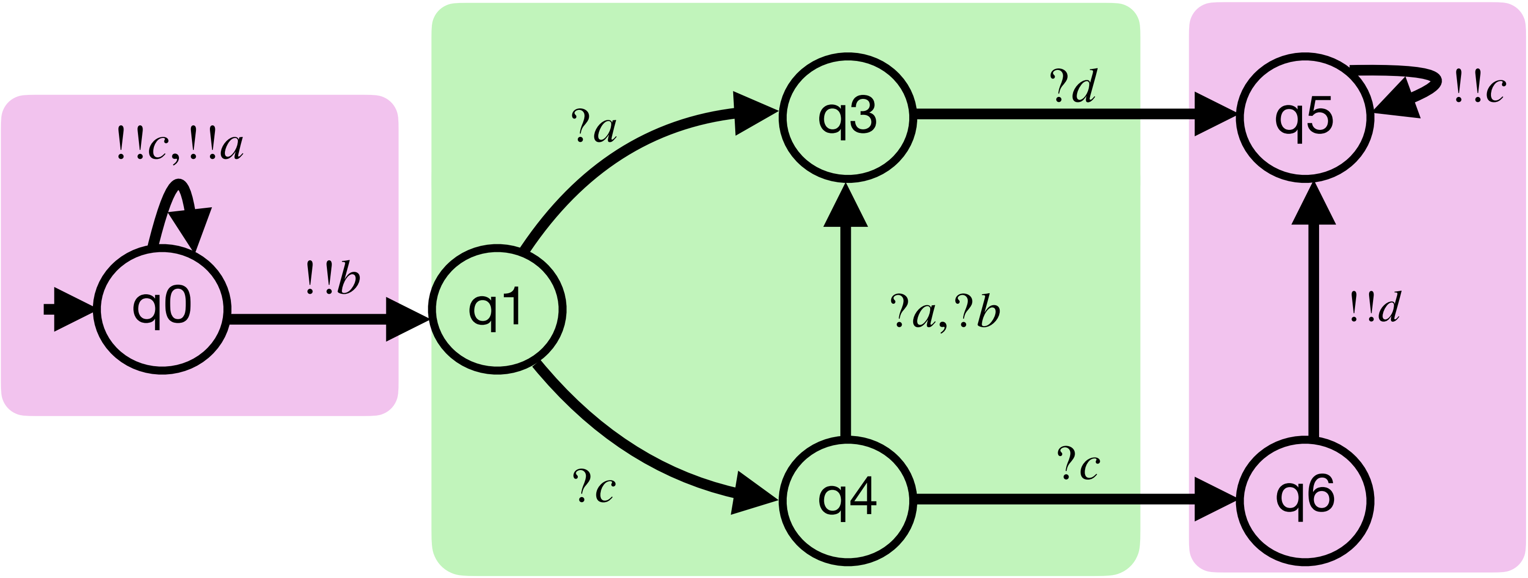
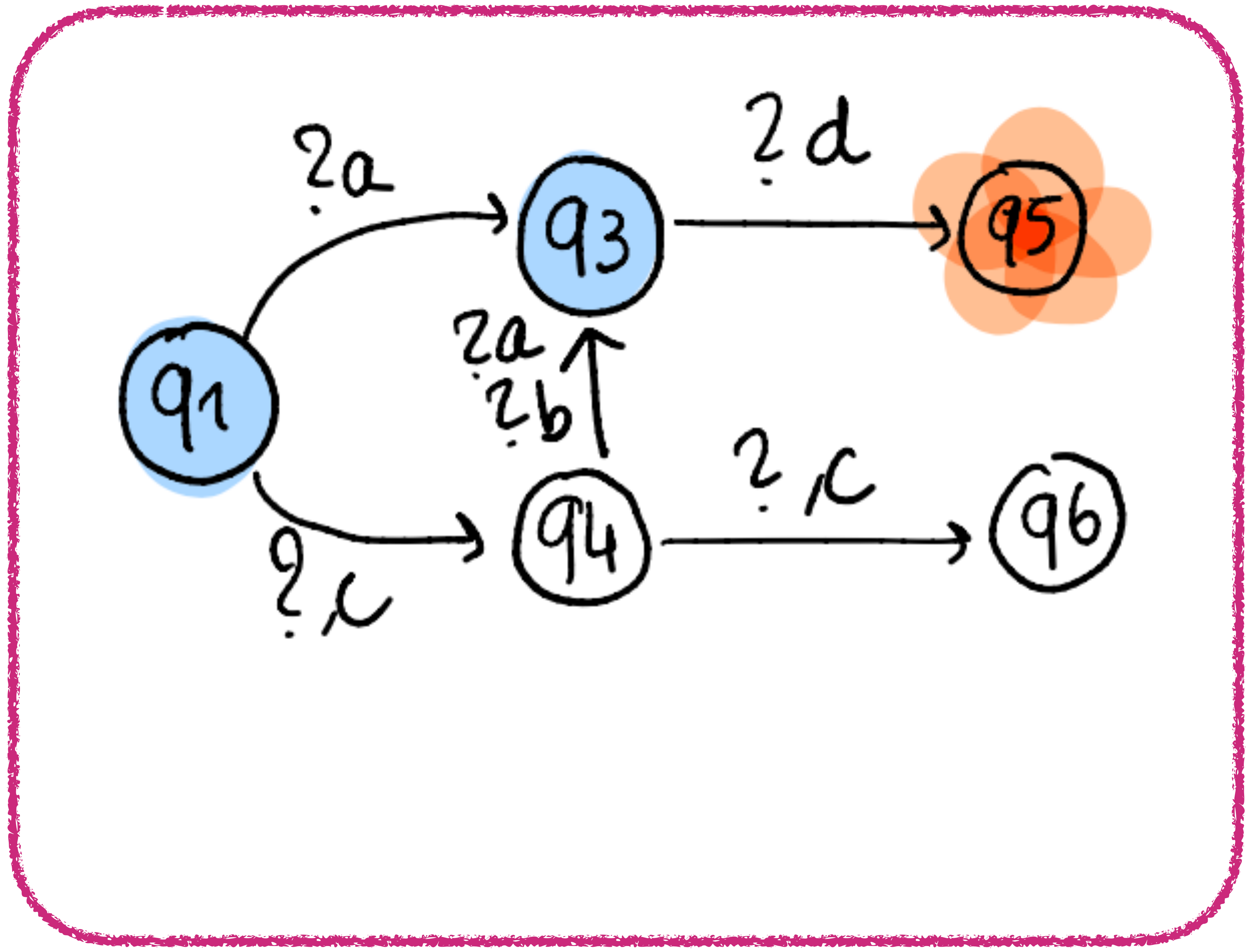


+ one counter

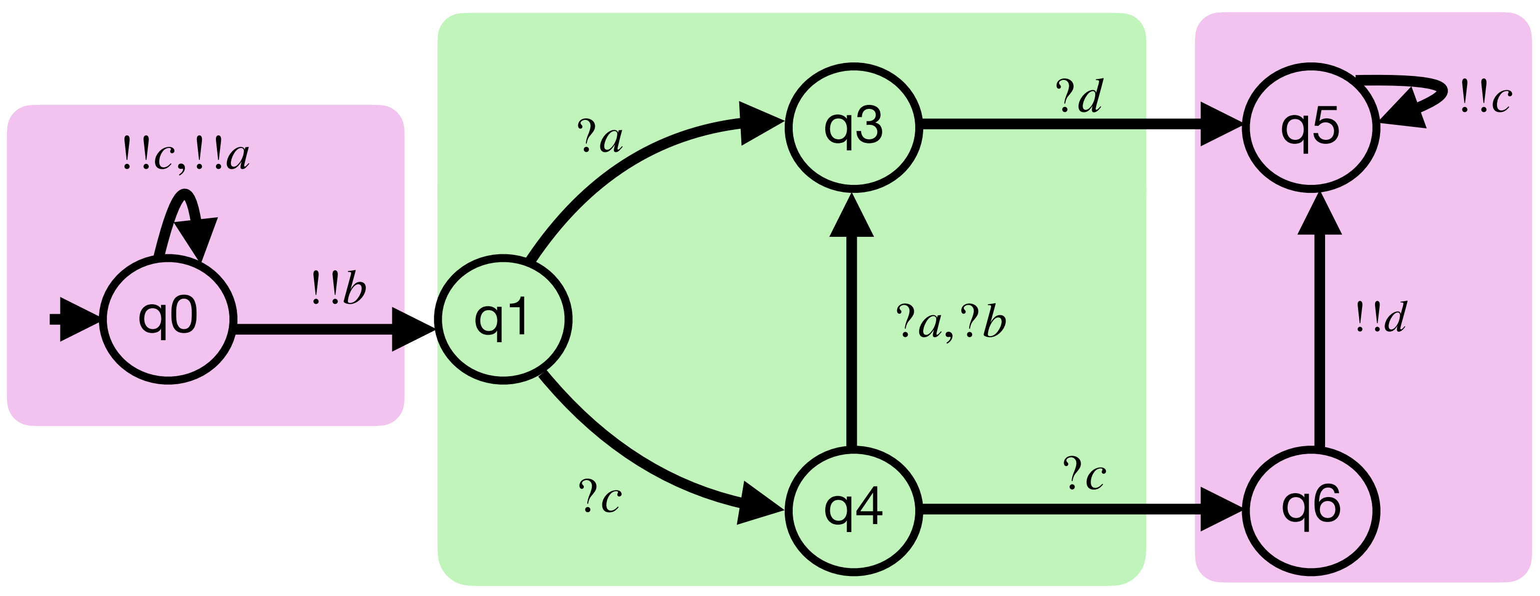


A Summary

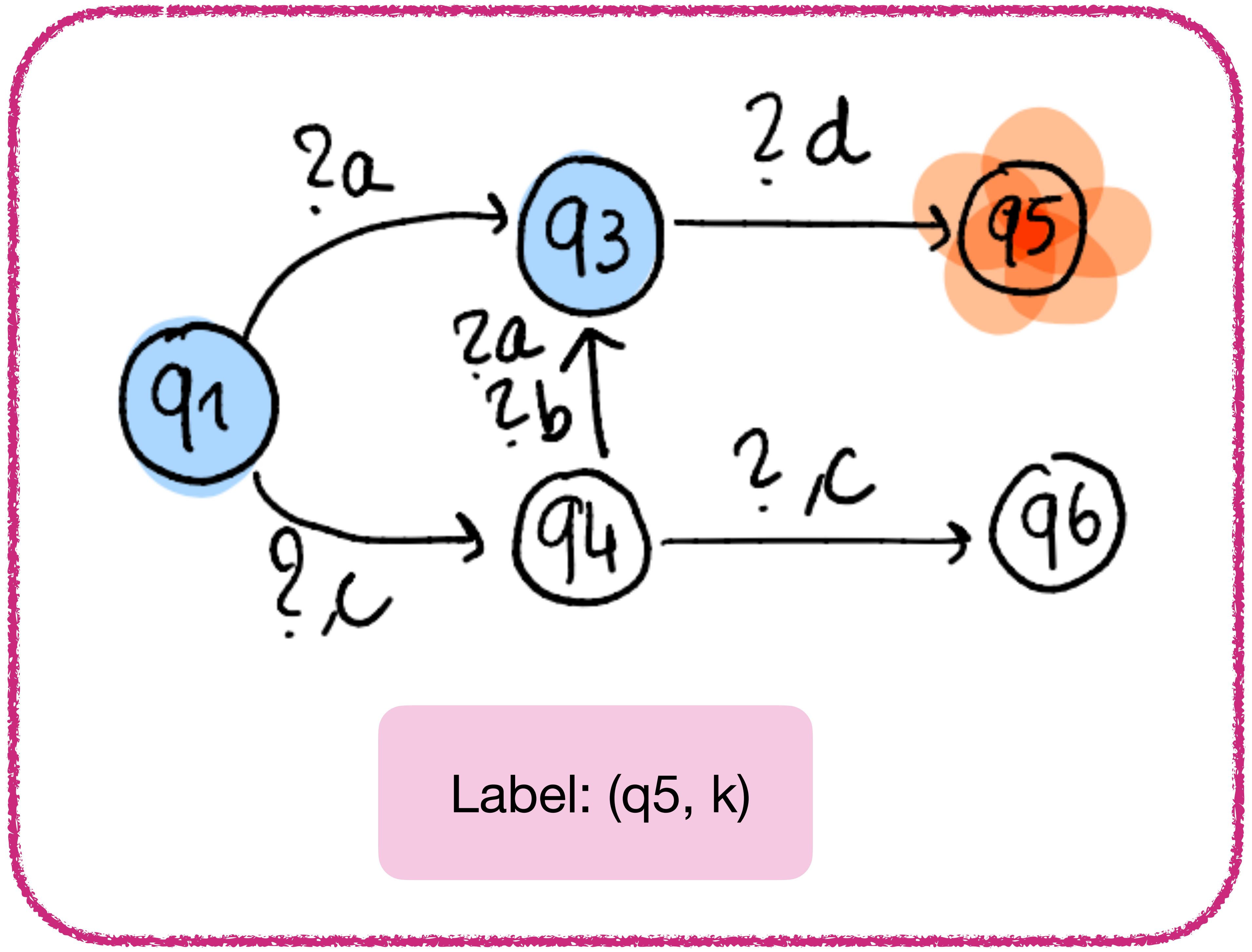


+ one counter

- Some processes are present on q1 and q3,
- the next action state they will reach is q5 and
- they will reach q5 at the same time



A Summary



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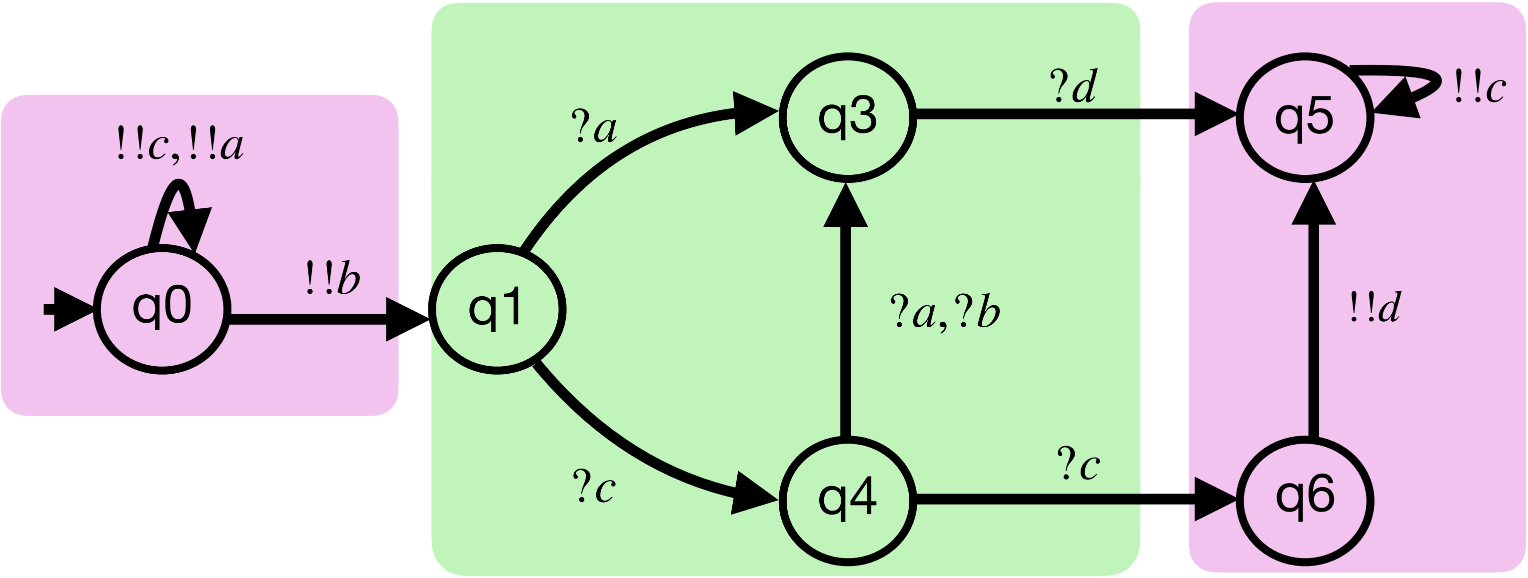
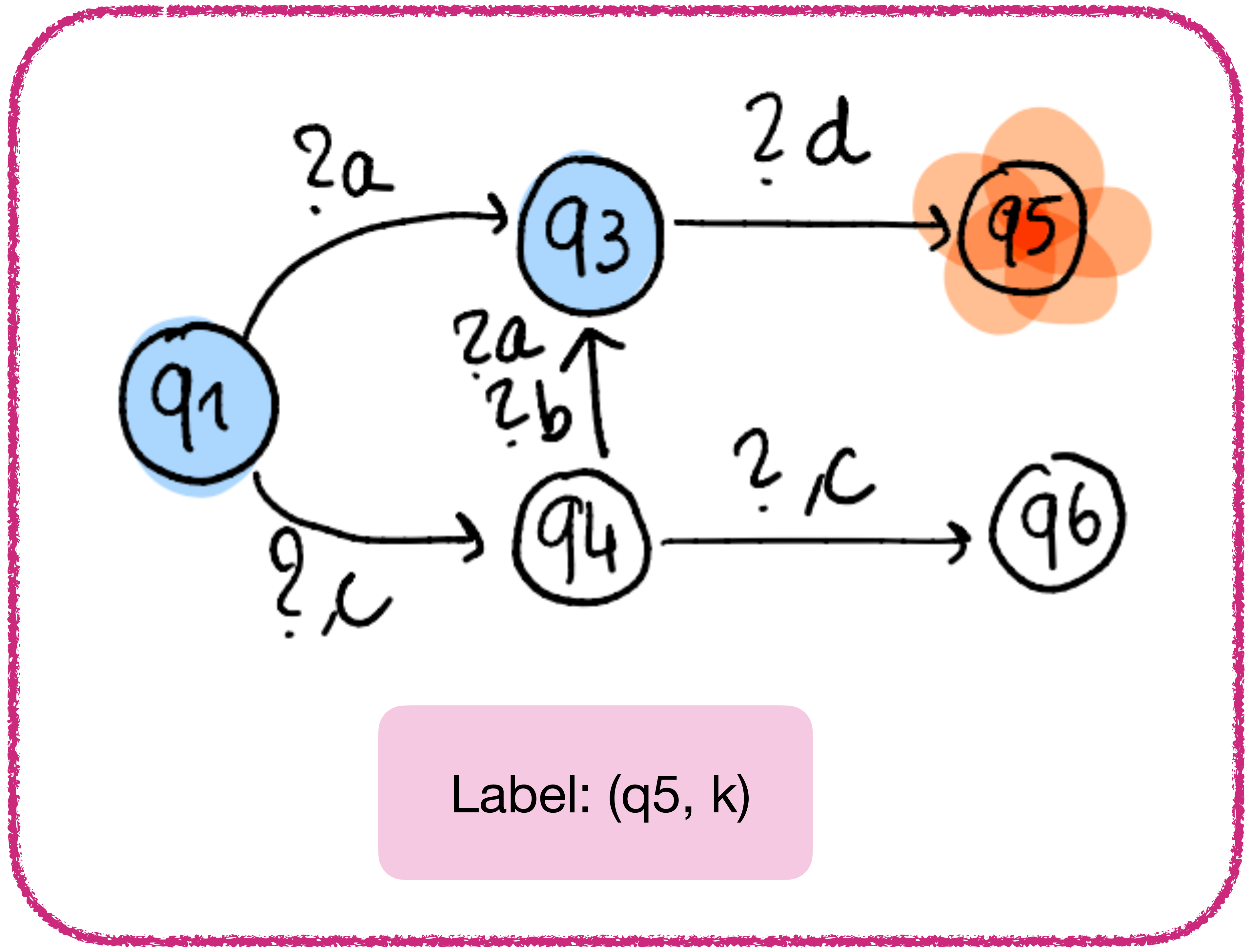


A Summary



$1 \leq k \leq$ #(waiting states)

+ one counter

- Some processes are present on q1 and q3,
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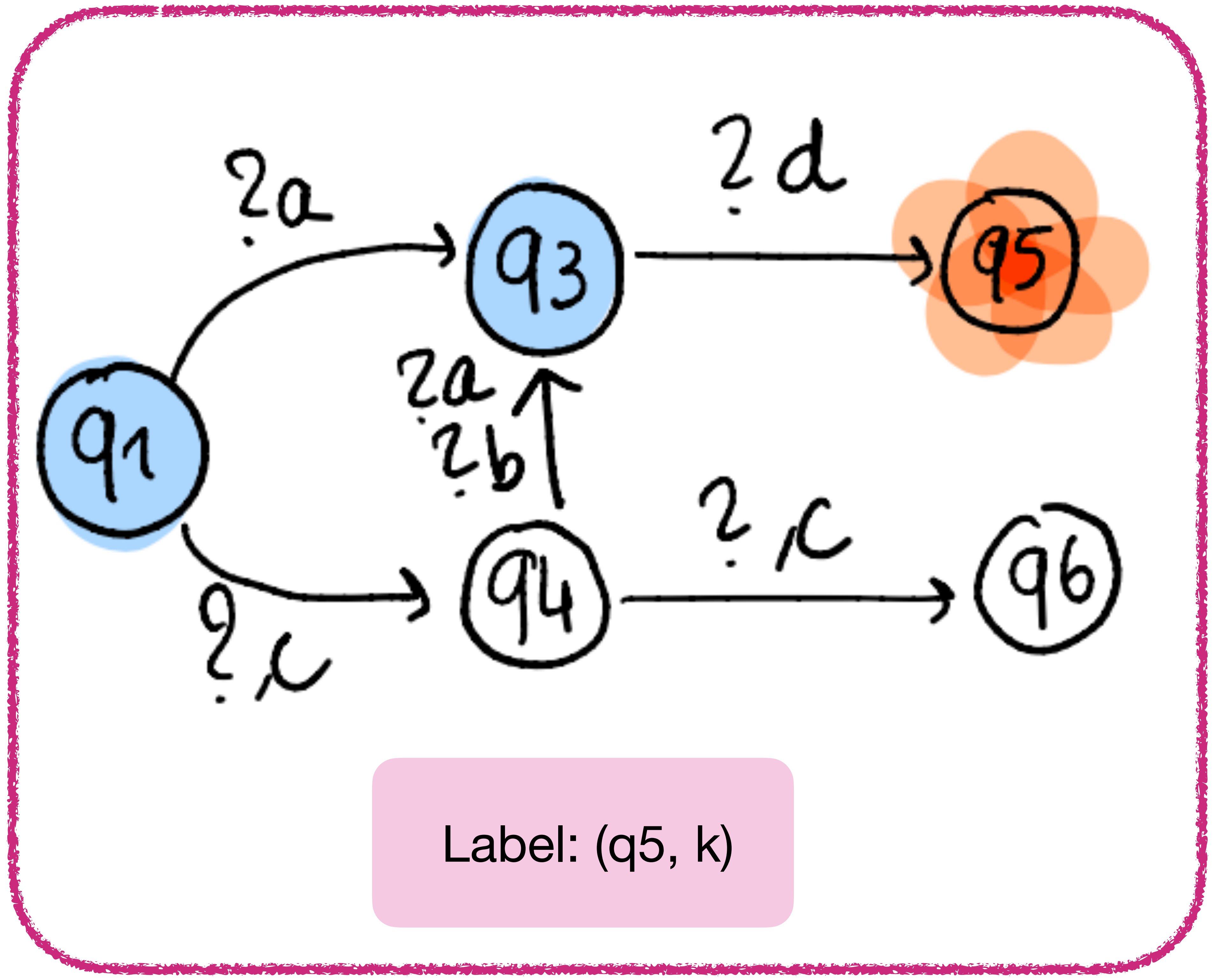
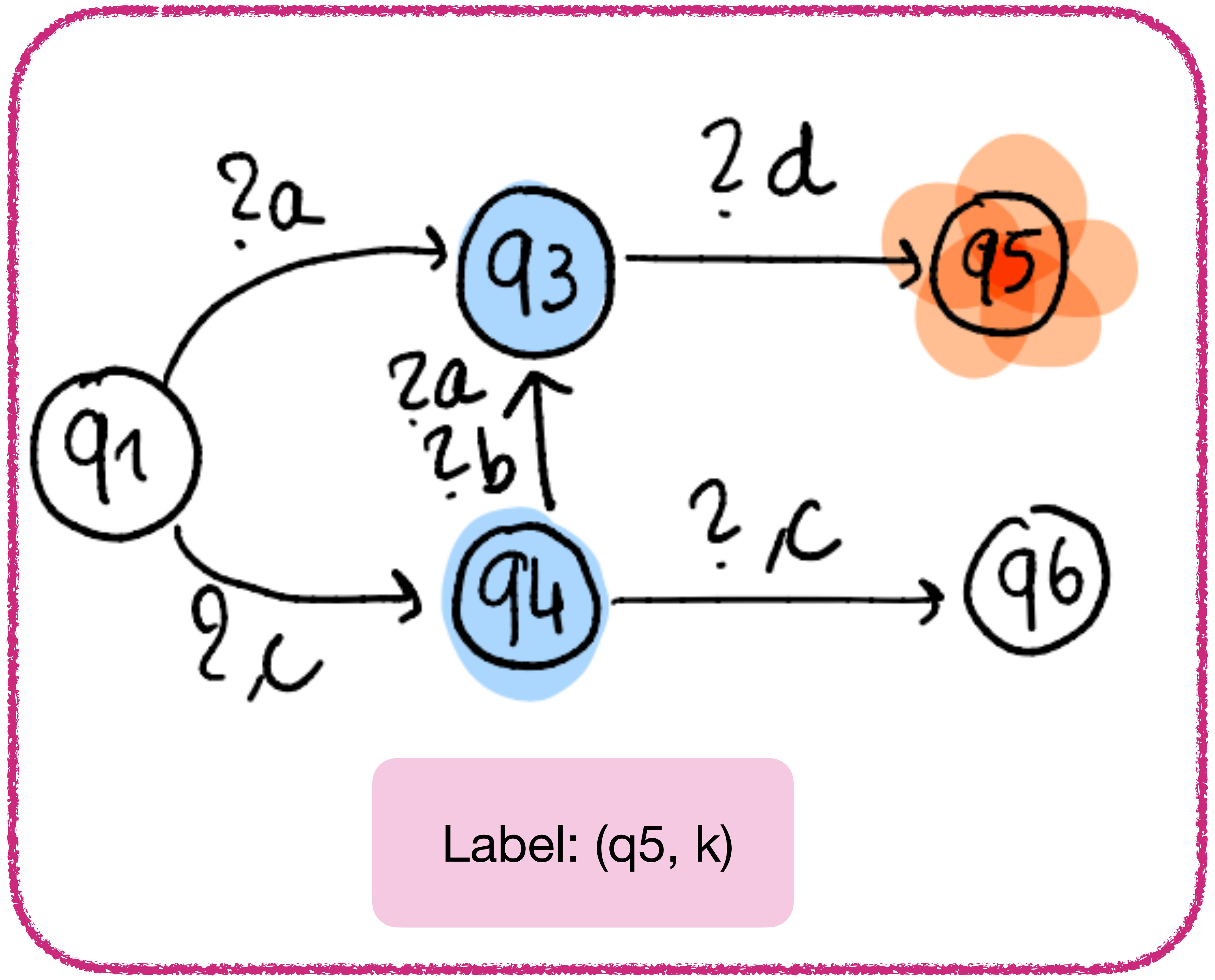
Broadcast and Summary

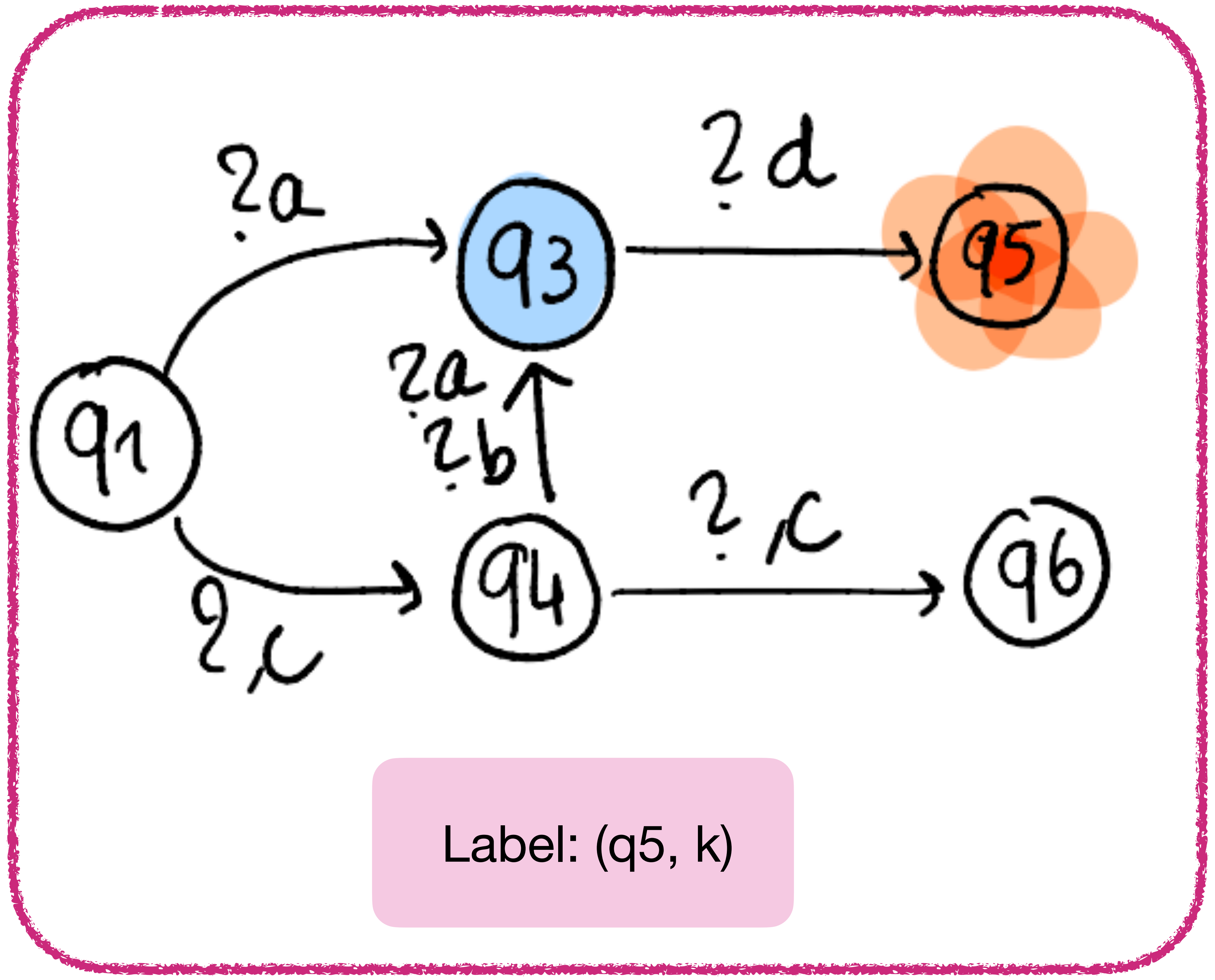


Broadcast and Summary



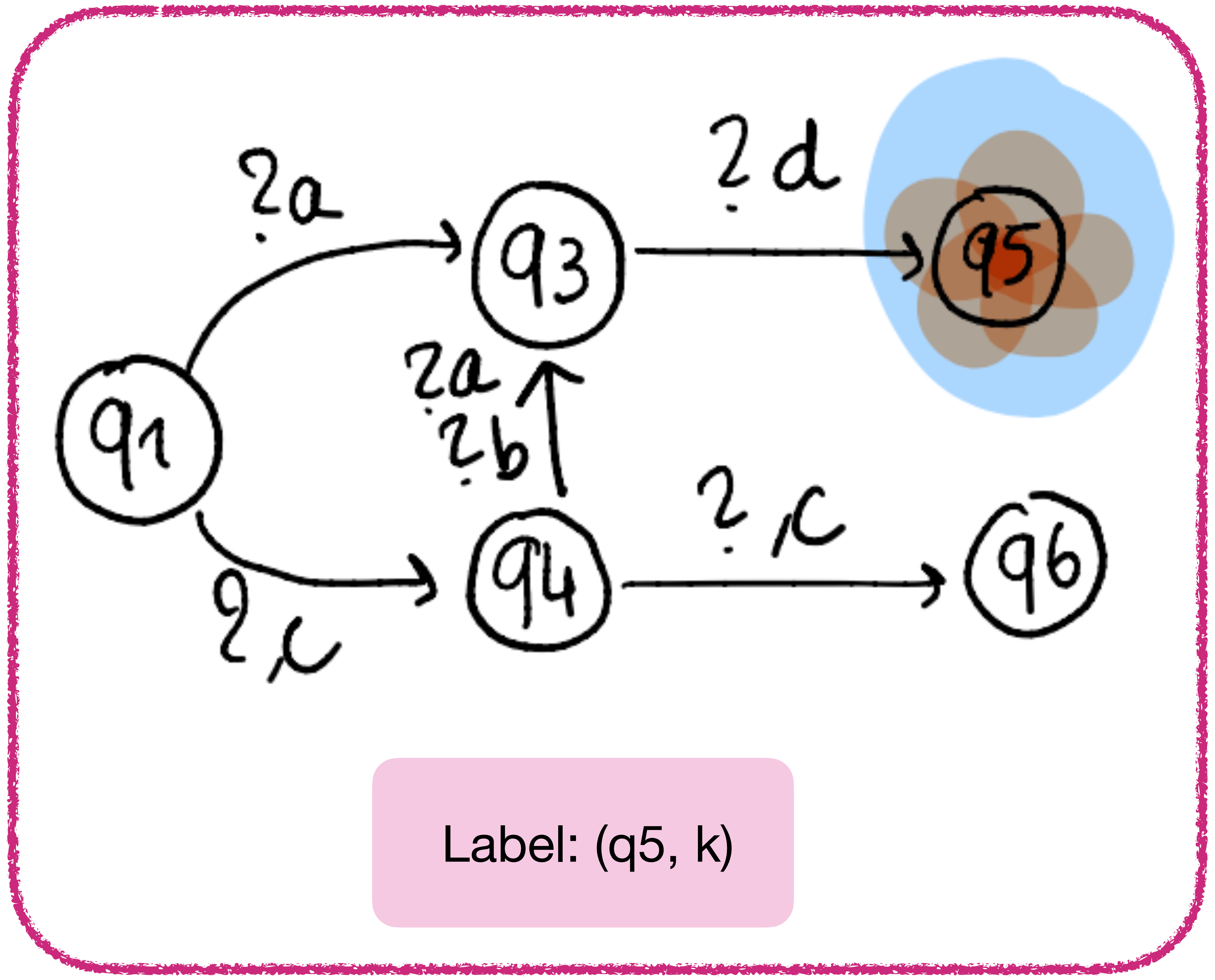
!!c

Broadcast and Summary



!!a

Broadcast and Summary

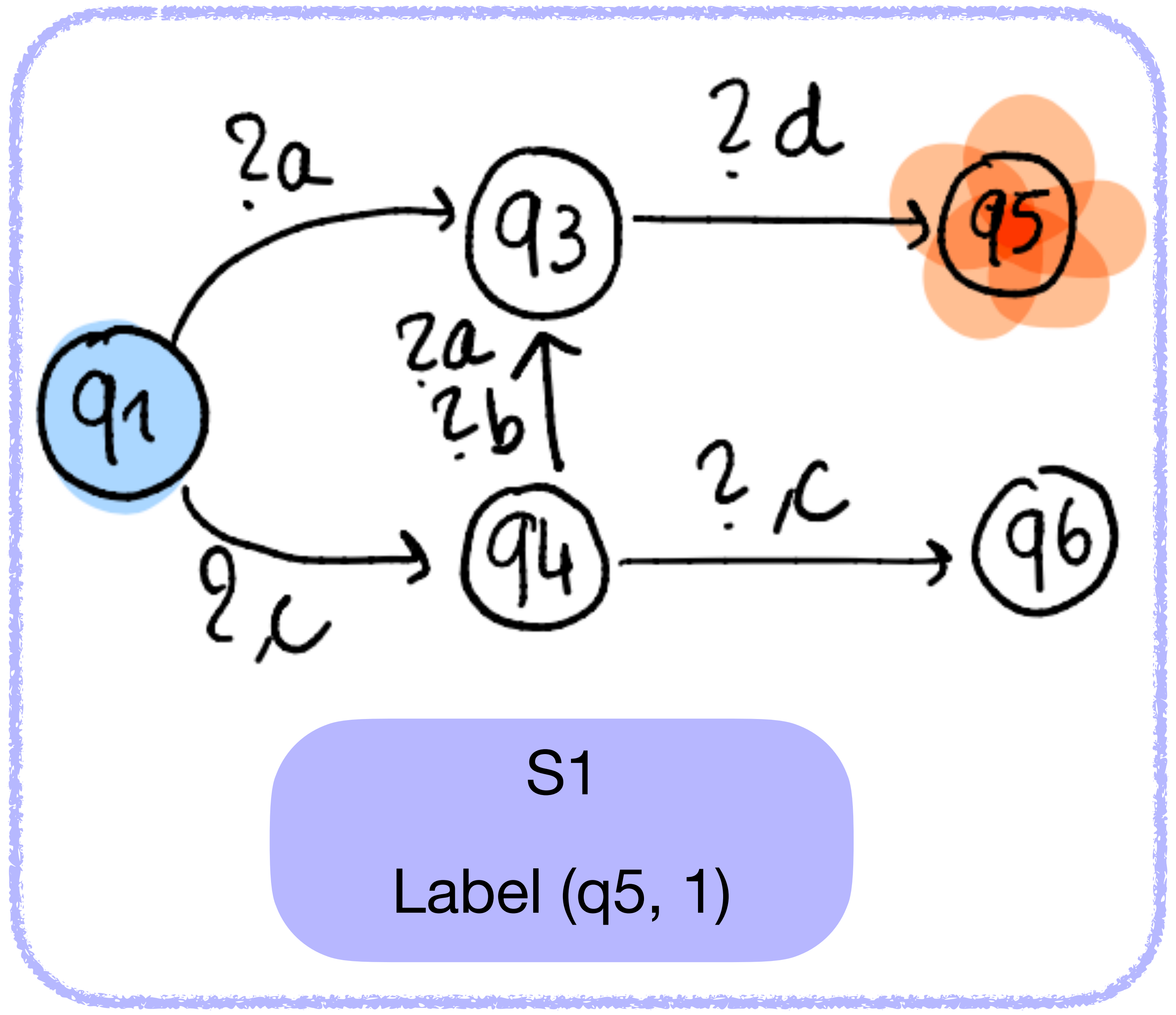


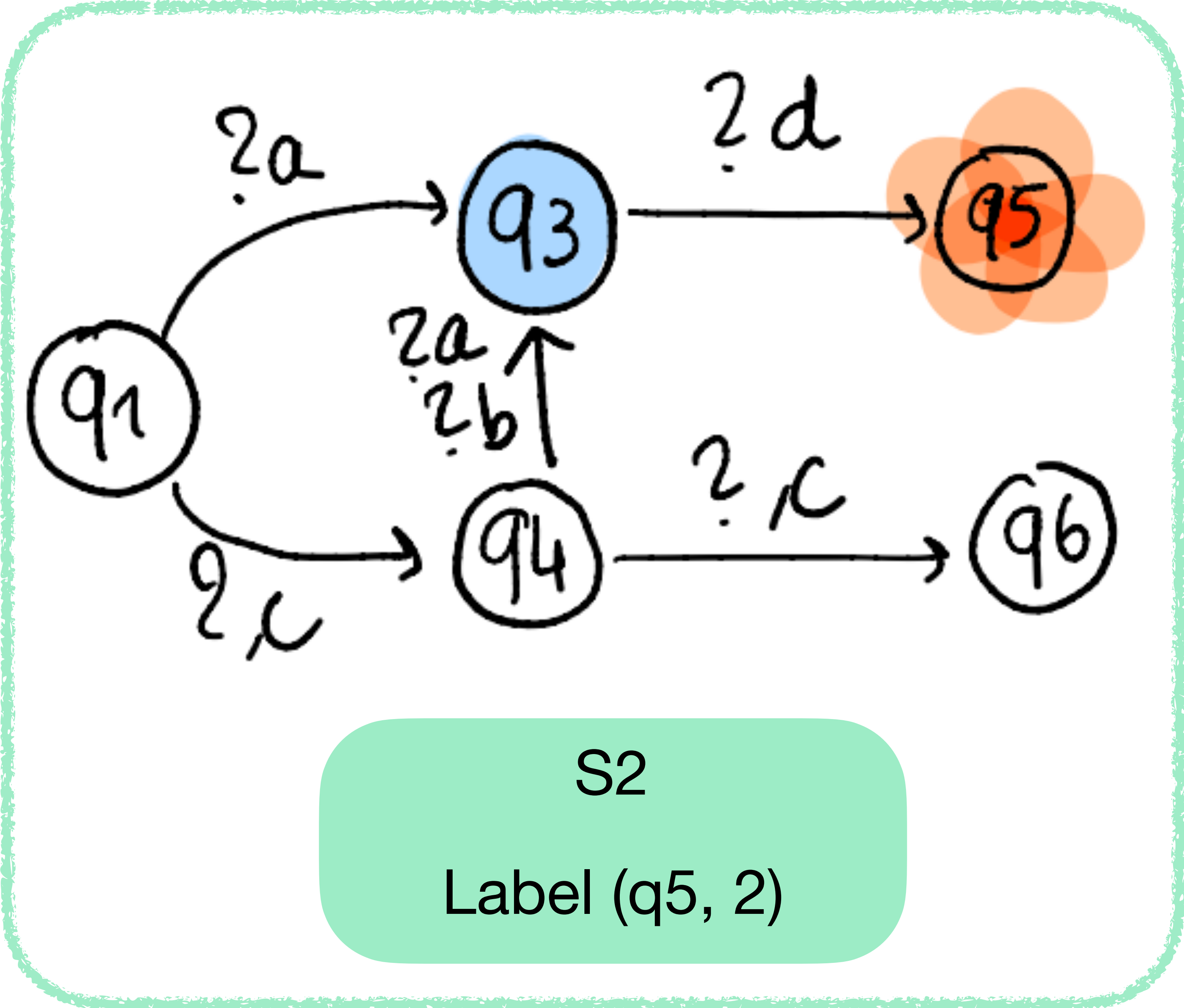
!!d

Summaries in VASS

- Location = **coherent** set of summaries
- Counters = one counter per action states + one counter per summary label
- In the VASS, we keep track of processes on action states, and guess some summaries for the processes on waiting states

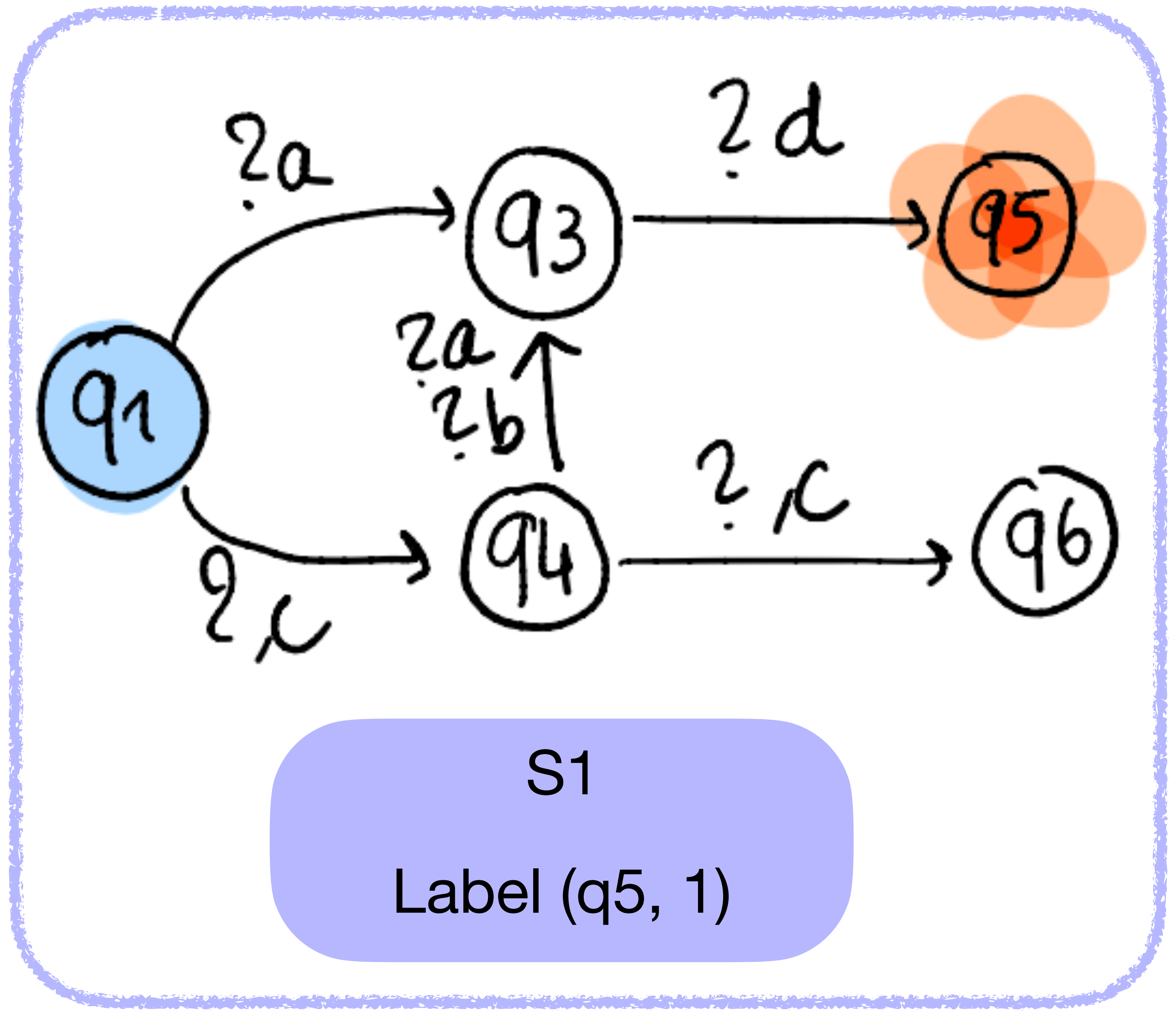
Coherent* sets of Summaries

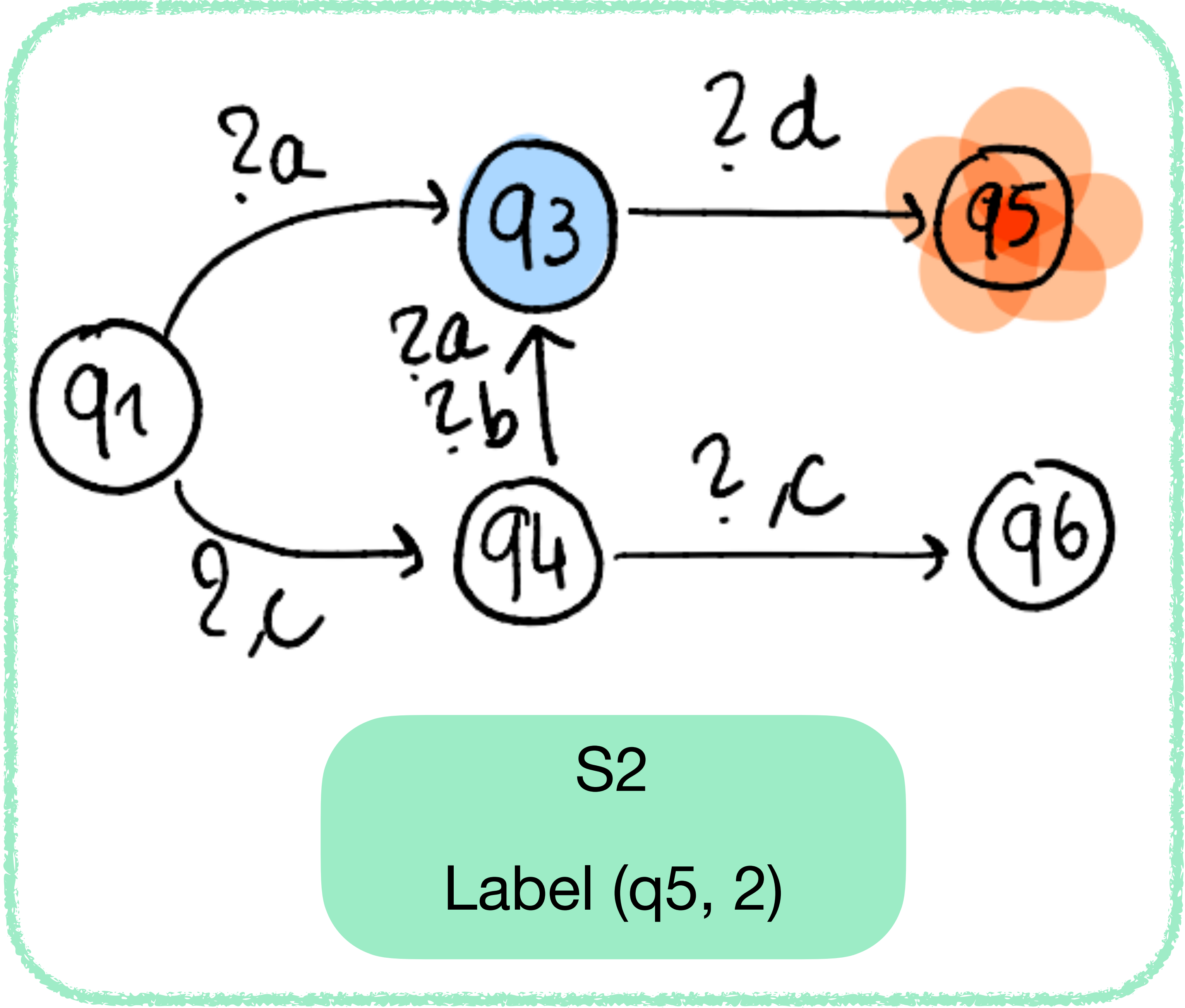




*** two processes on different summaries don’t reach the same state OR reach the same state but not at the same time**

Coherent* sets of Summaries





ex: !!d !!a !!d

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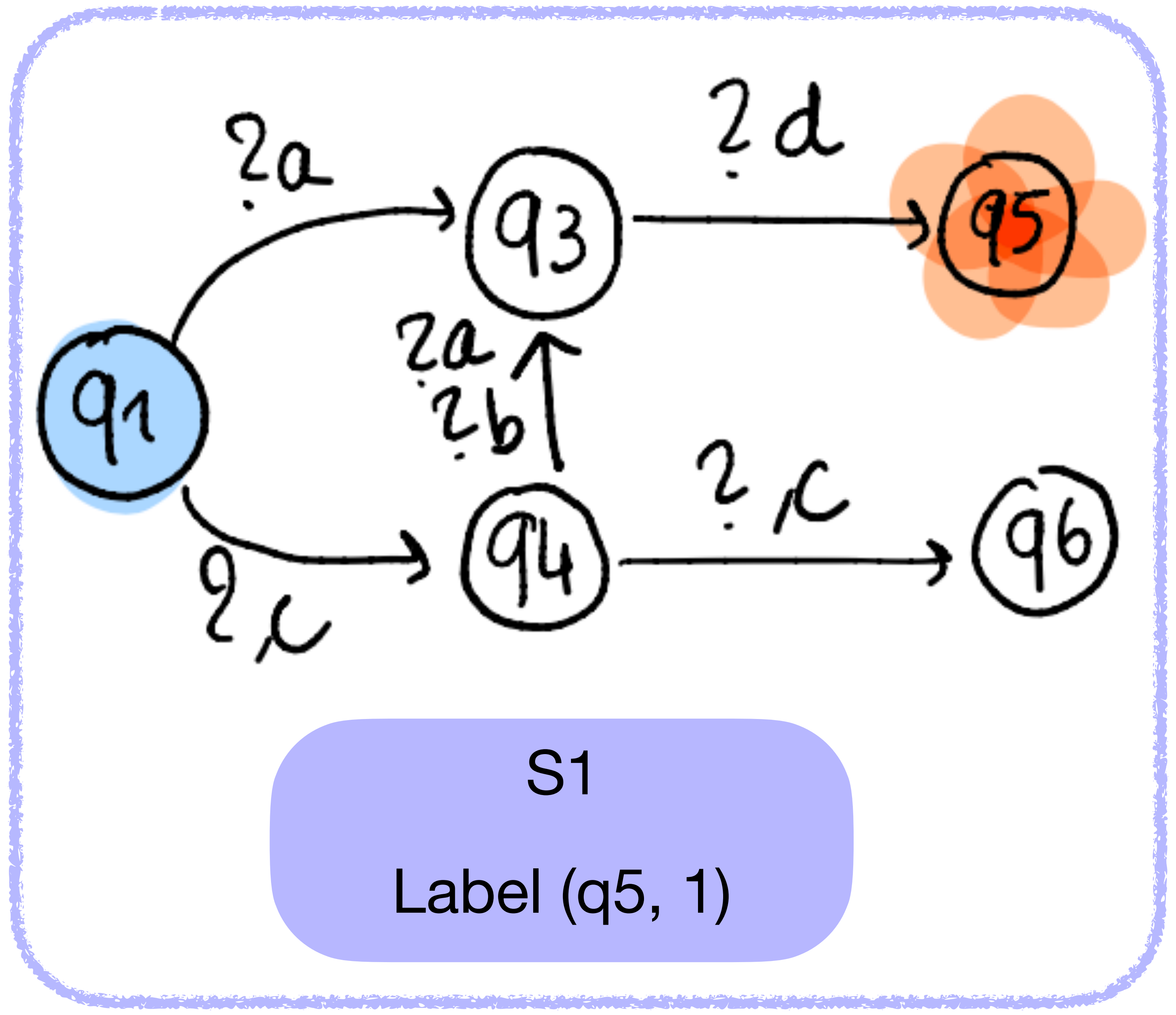
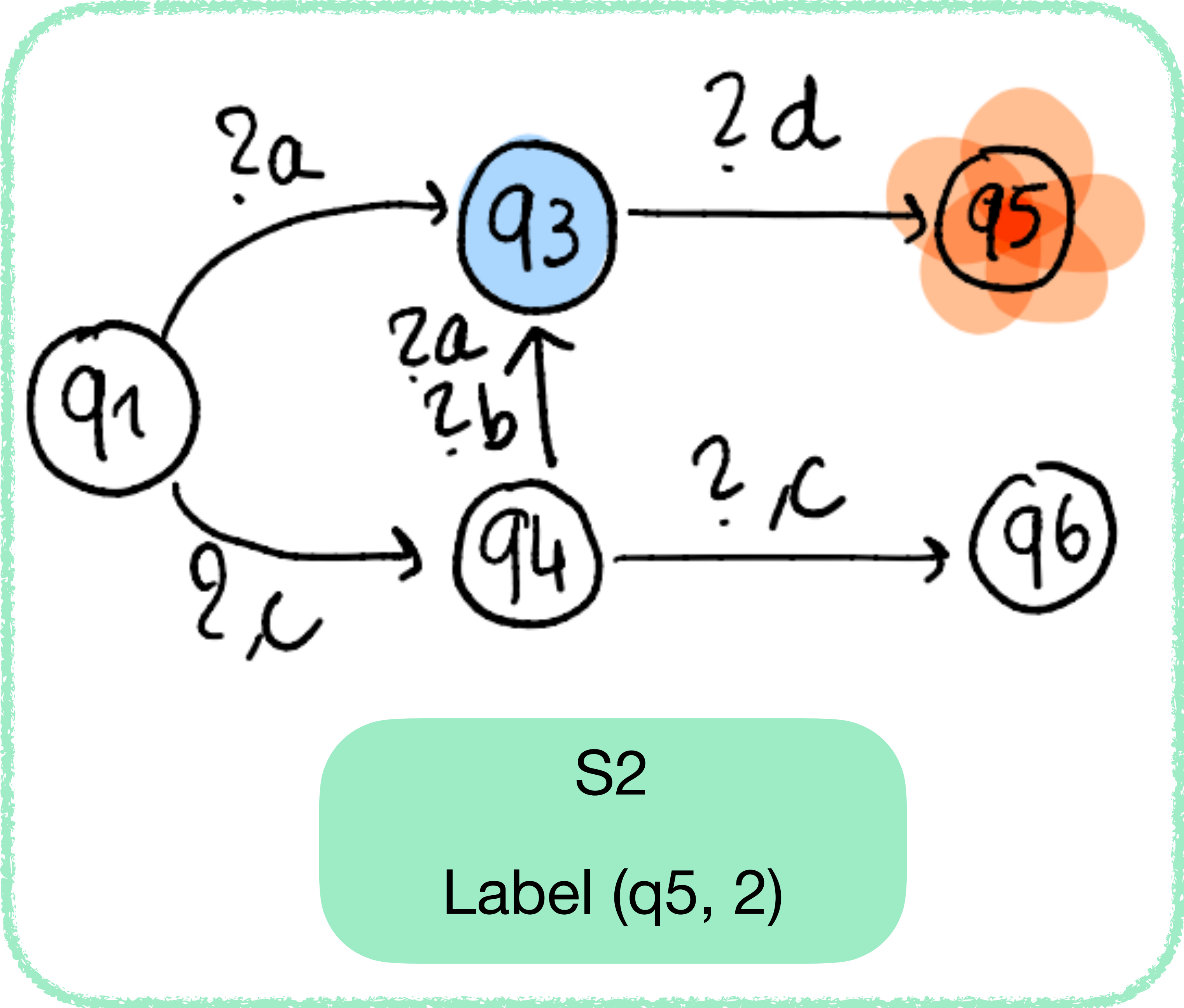
Coherent* sets of Summaries

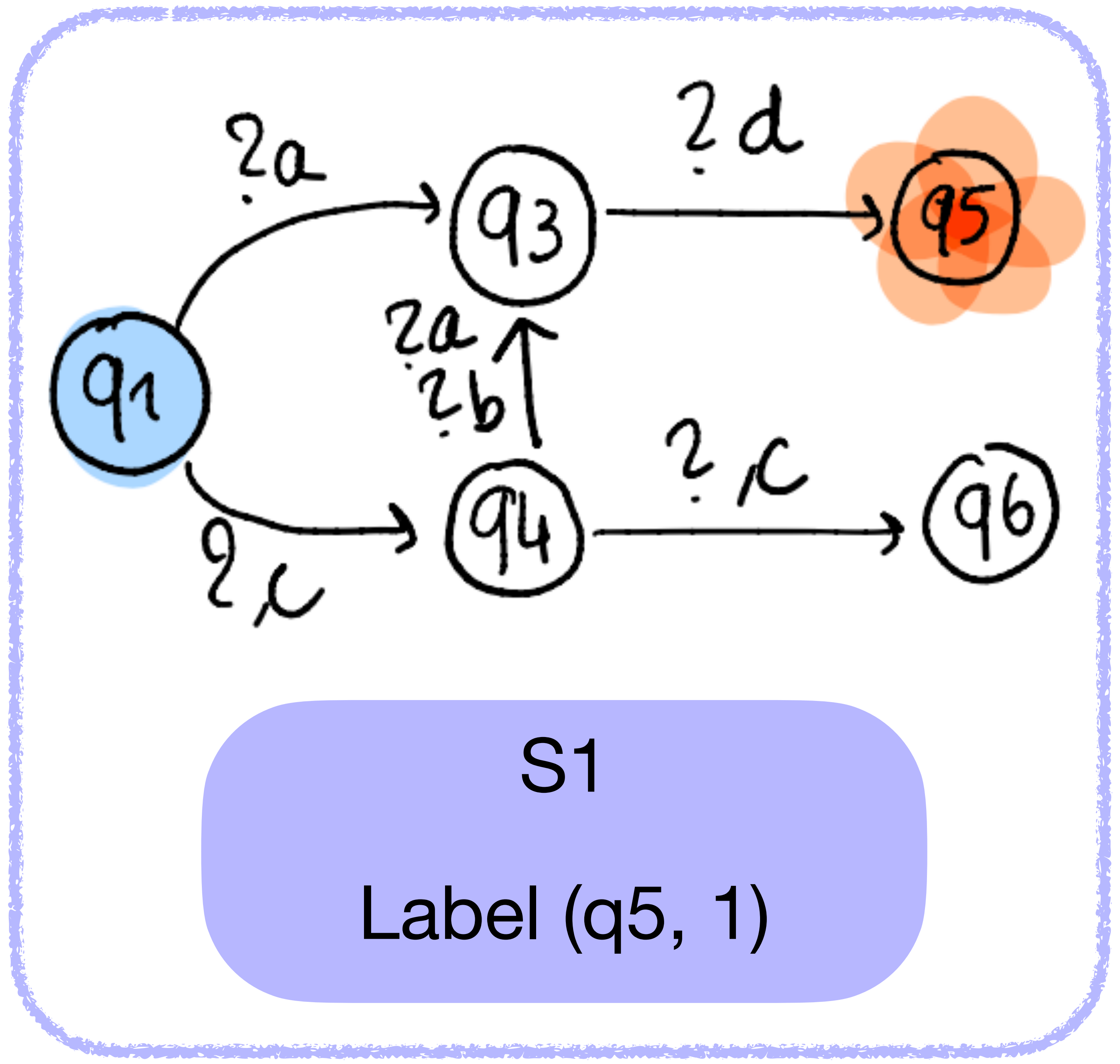


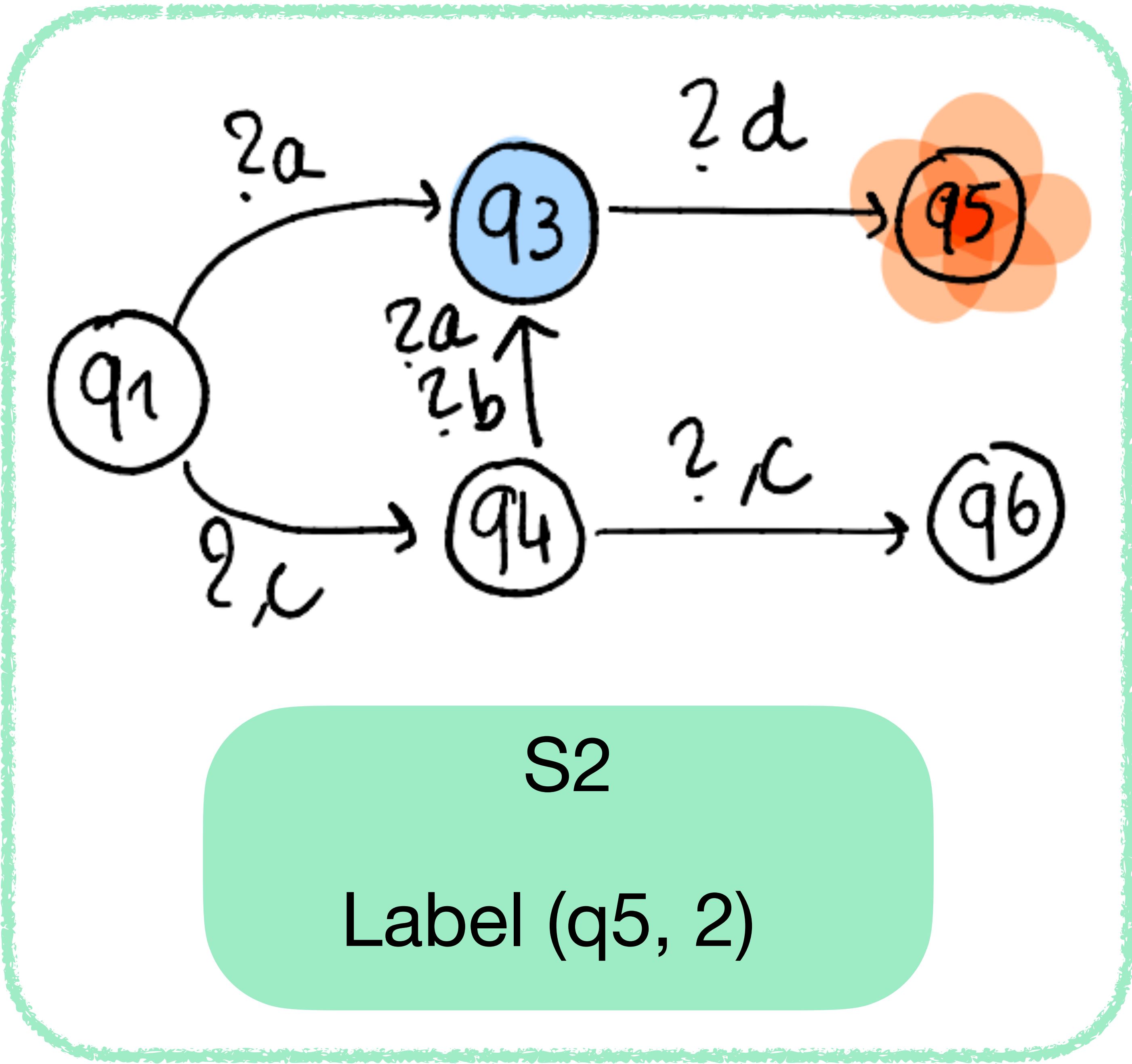


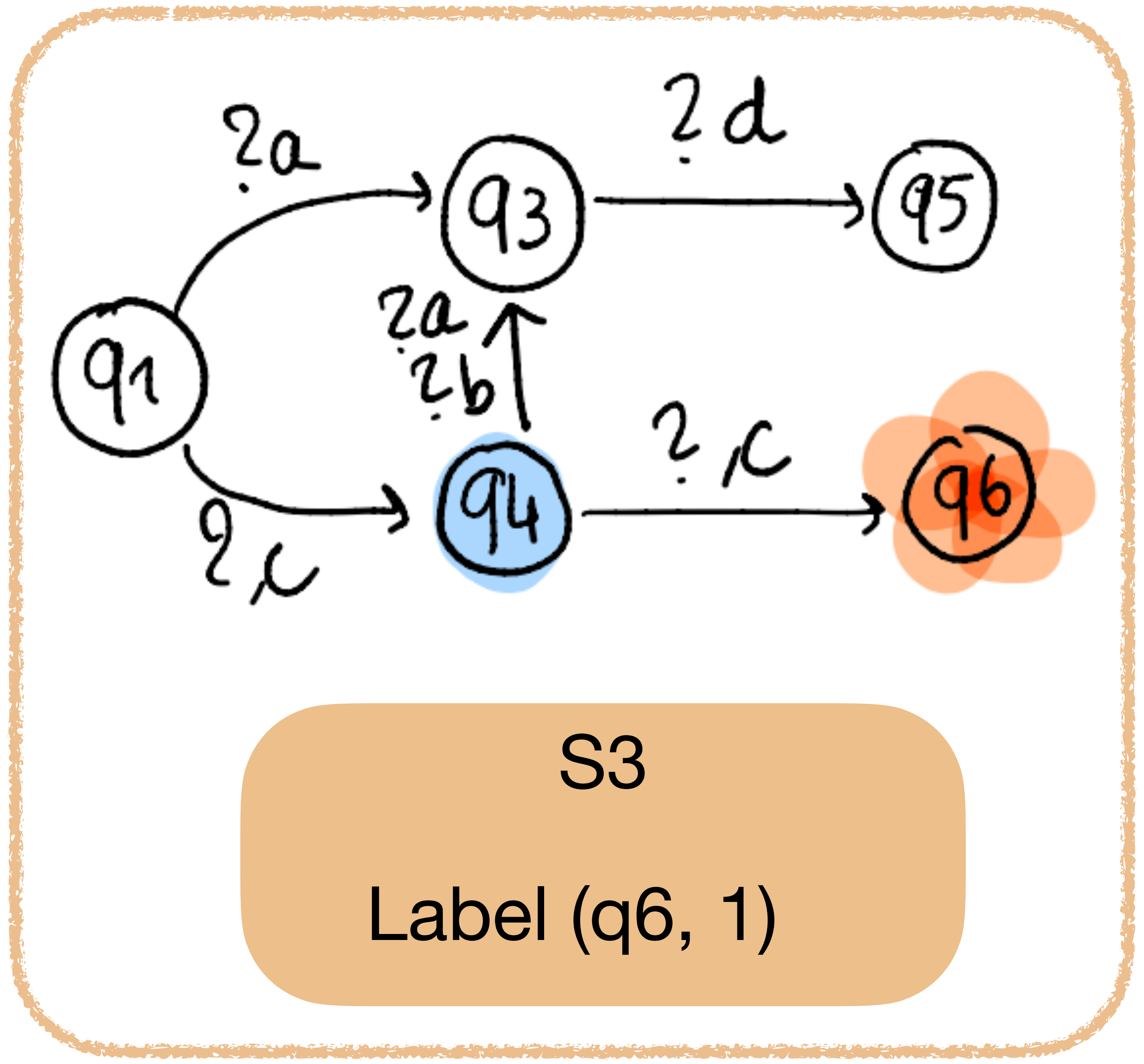
Coherent!

* two processes on dif... ...maries don’t reach the same state
OR reach the same state but not at the same time

Coherent* sets of Summaries







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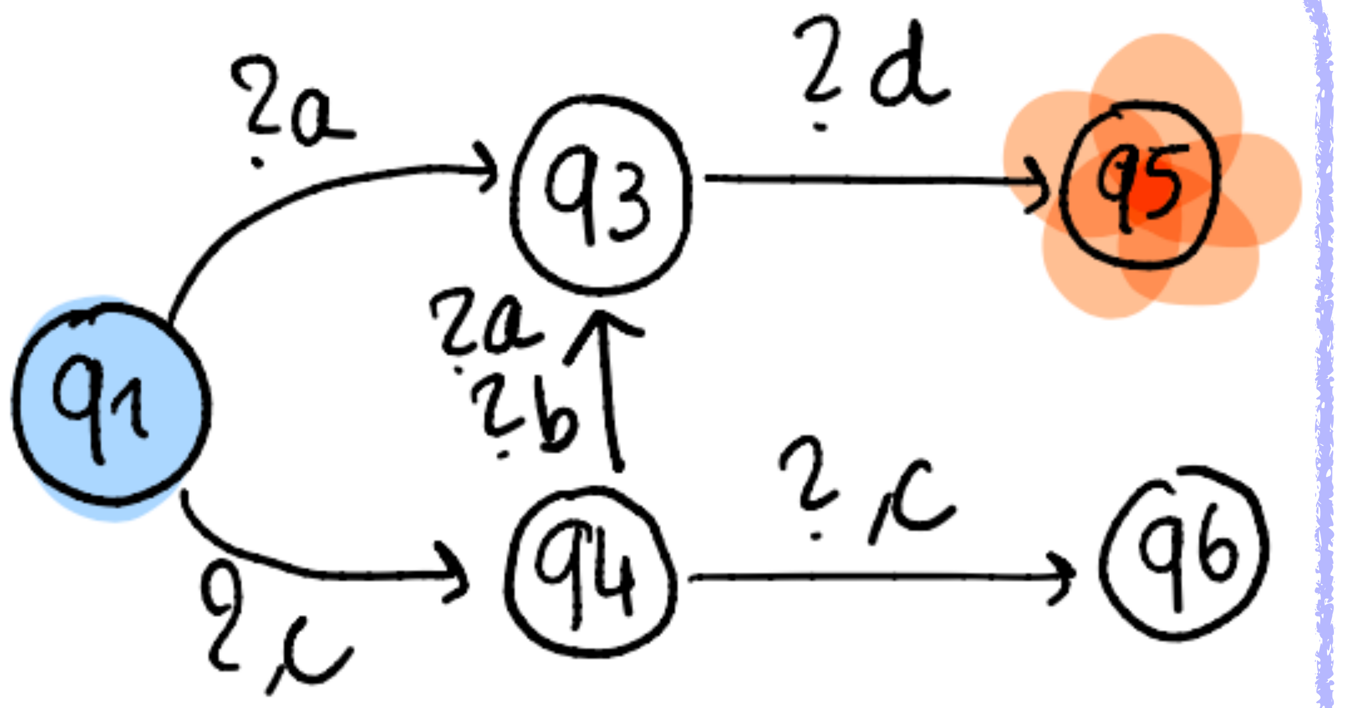
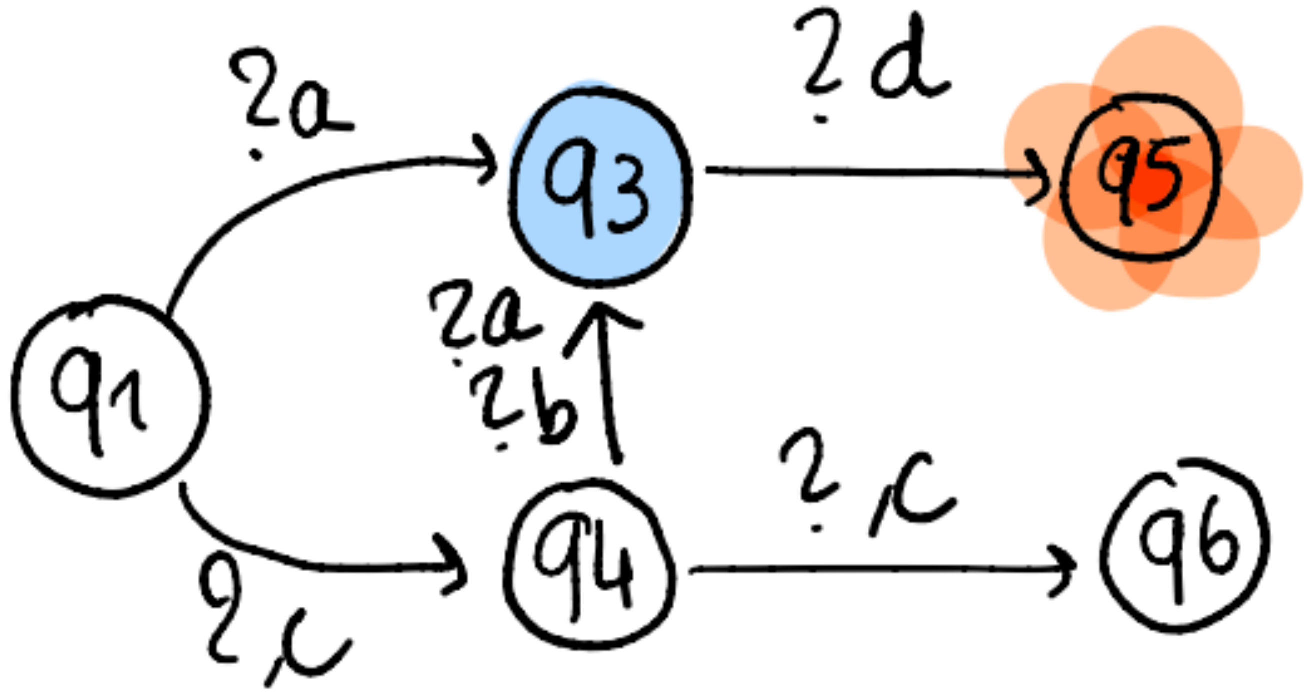
Coherent* sets of Summaries



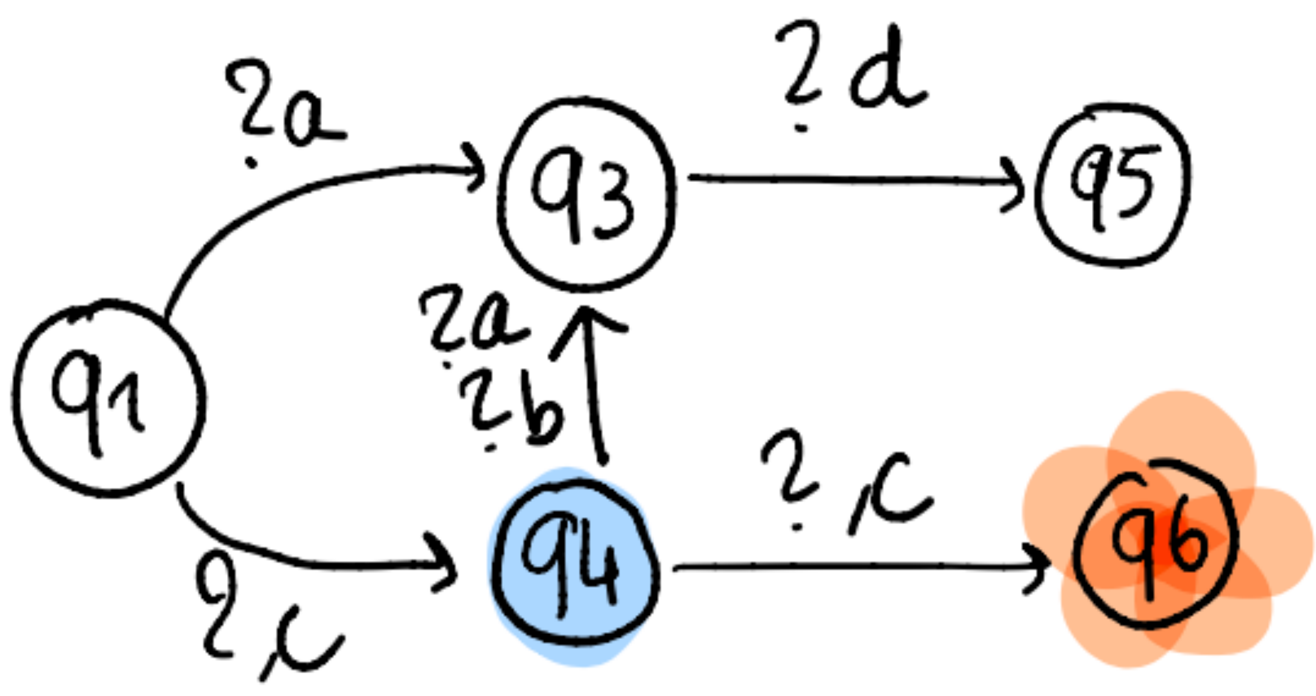
S1

Label (q5, 1)



S2

Label (q5, 2)



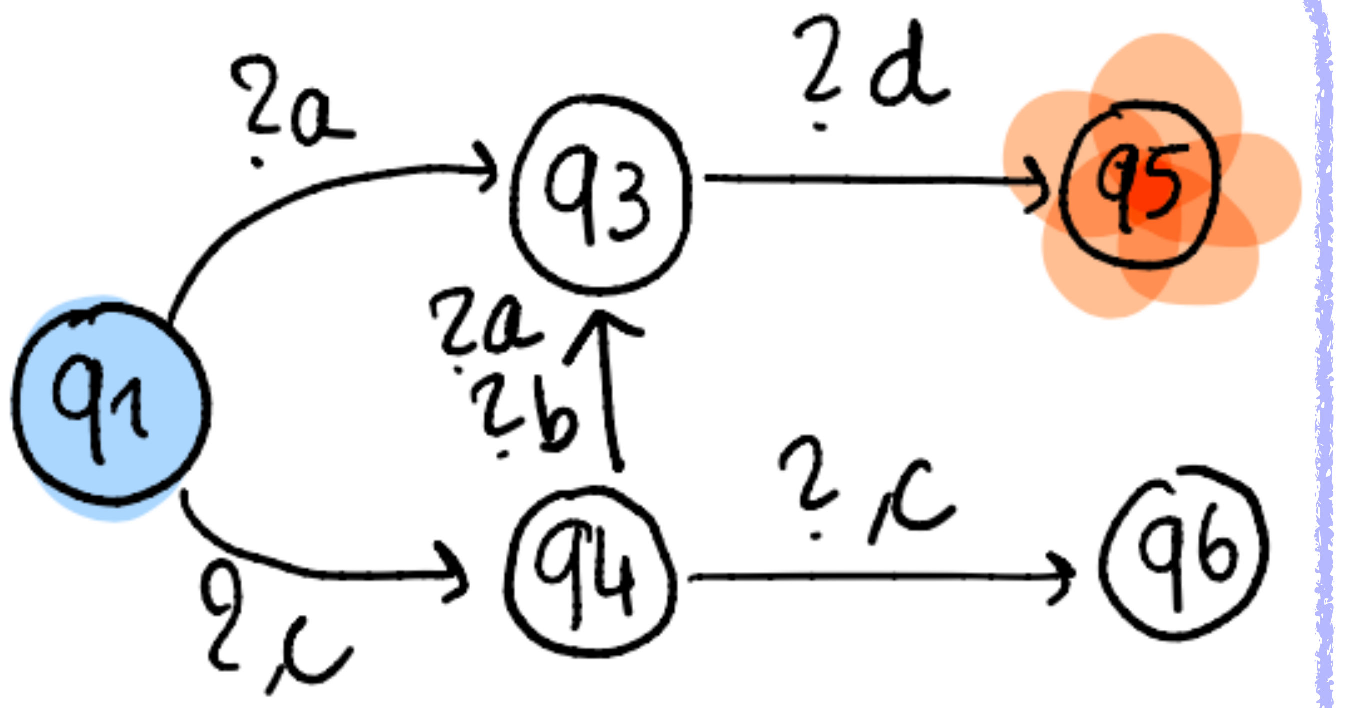
S3

Label (q6, 1)

ex: !!d !!c !!b !!d

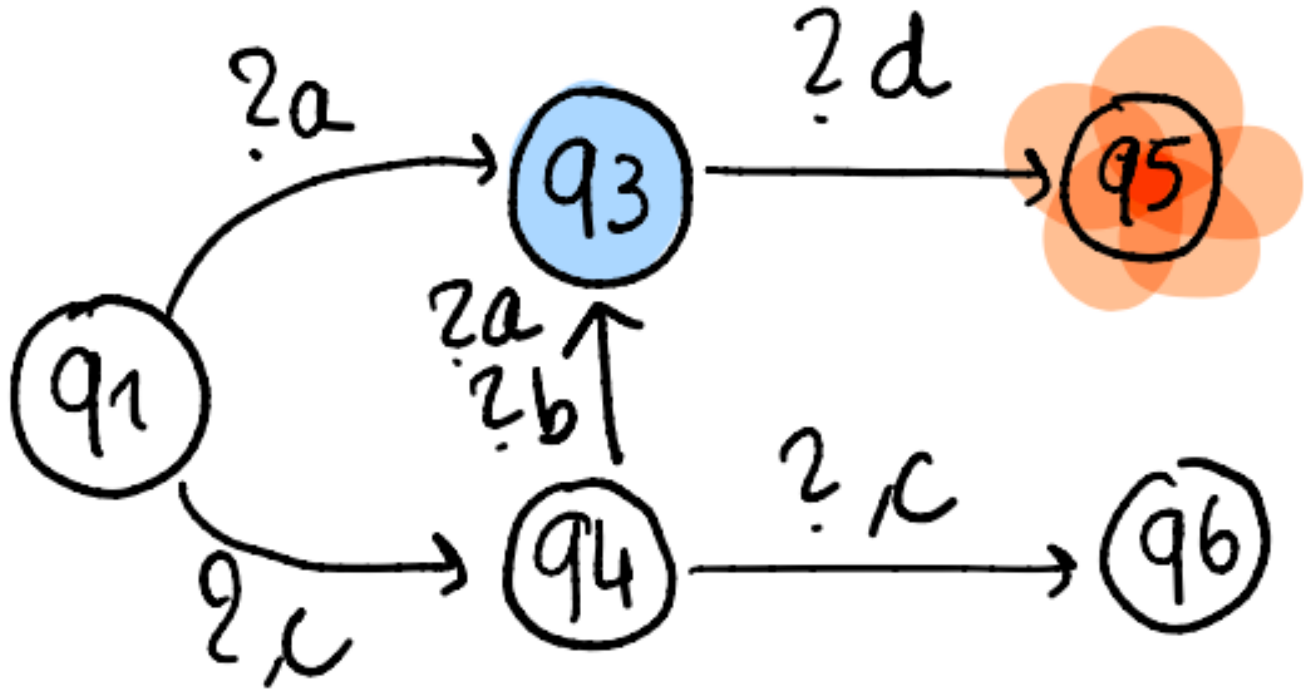
*** two processes on different summaries don't reach the same state OR reach the same state but not at the same time**

Coherent* sets of Summaries



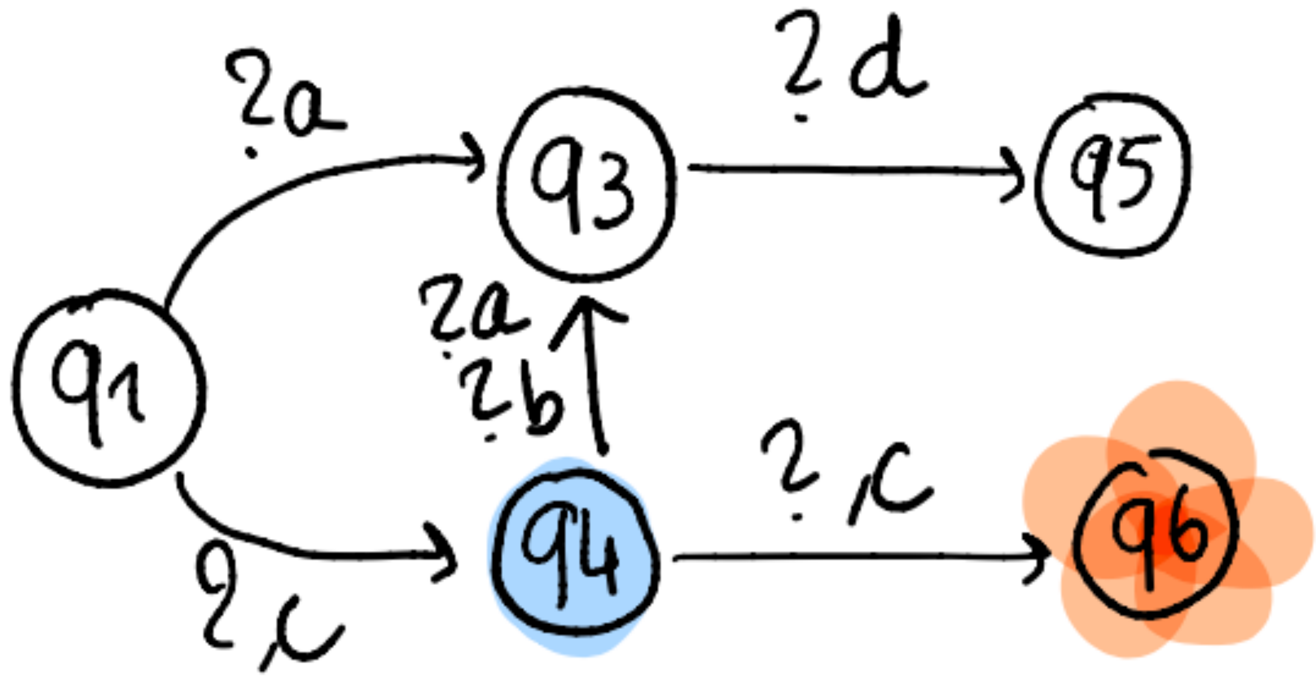
S1

Label (q5, 1)



S2

Label (q5, 2)



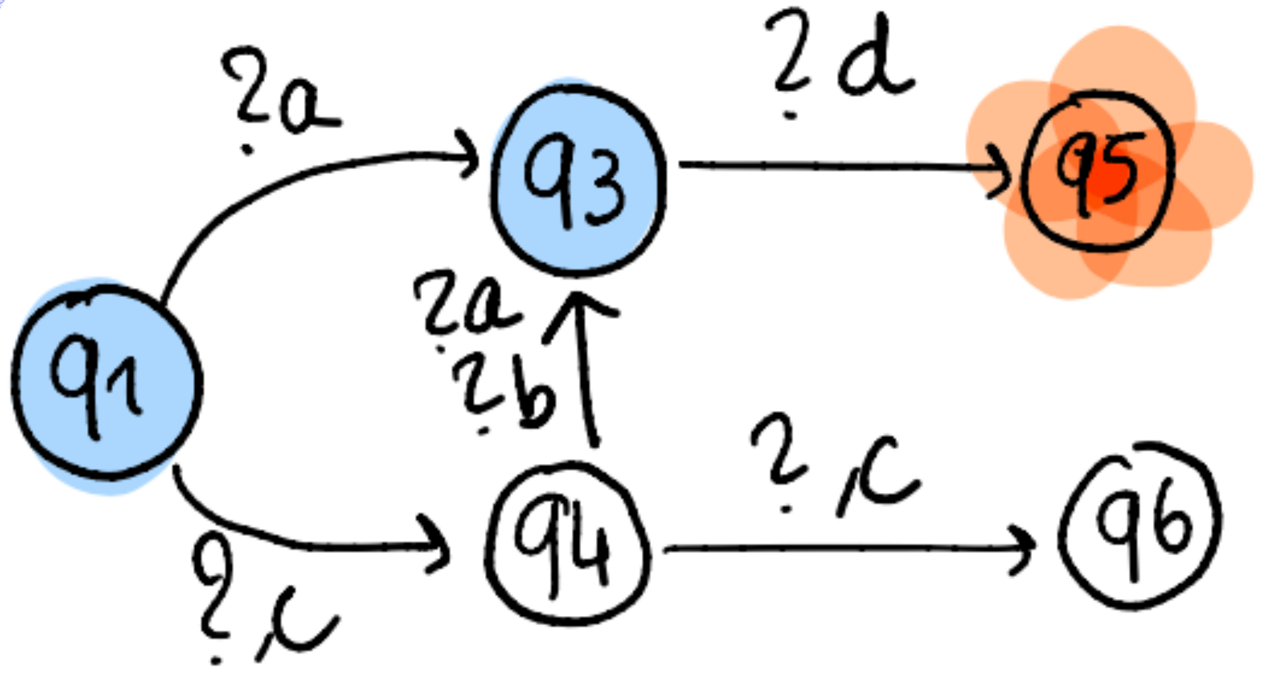
S3

Label (q6, 1)

Coherent again!

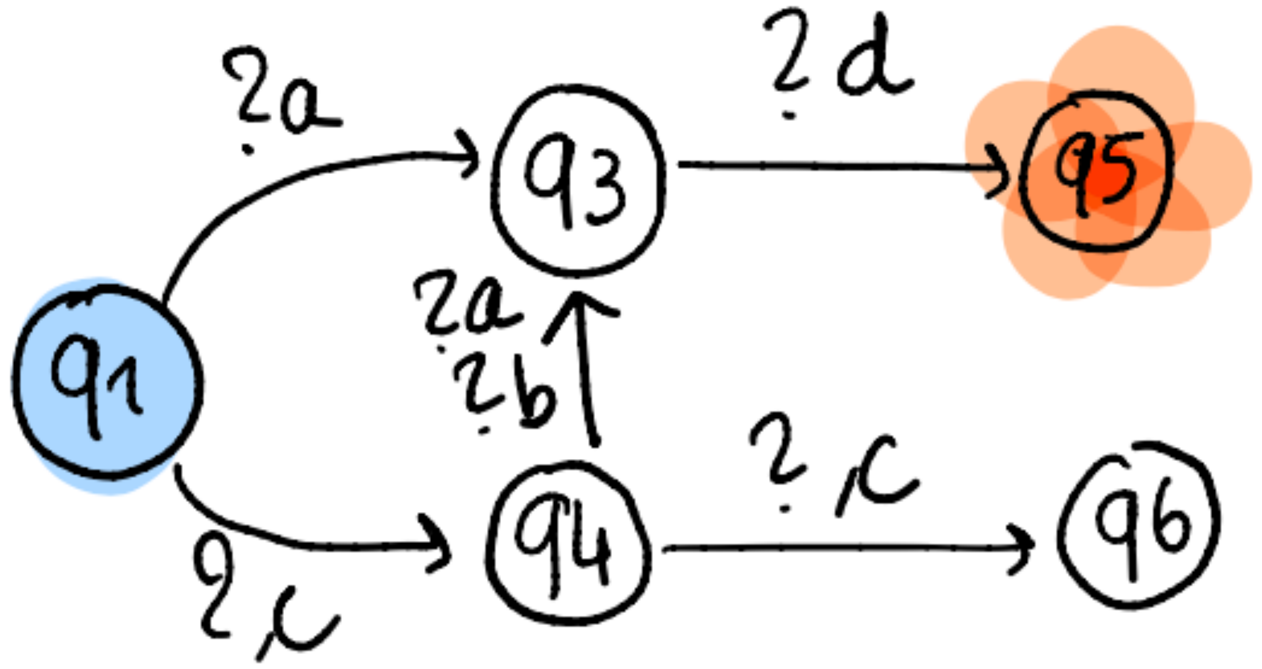
* two processes on different summaries don't reach the same state OR reach the same state but not at the same time

Coherent sets of Summaries



S1

Label (q5, 1)



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Label (q5, 2)

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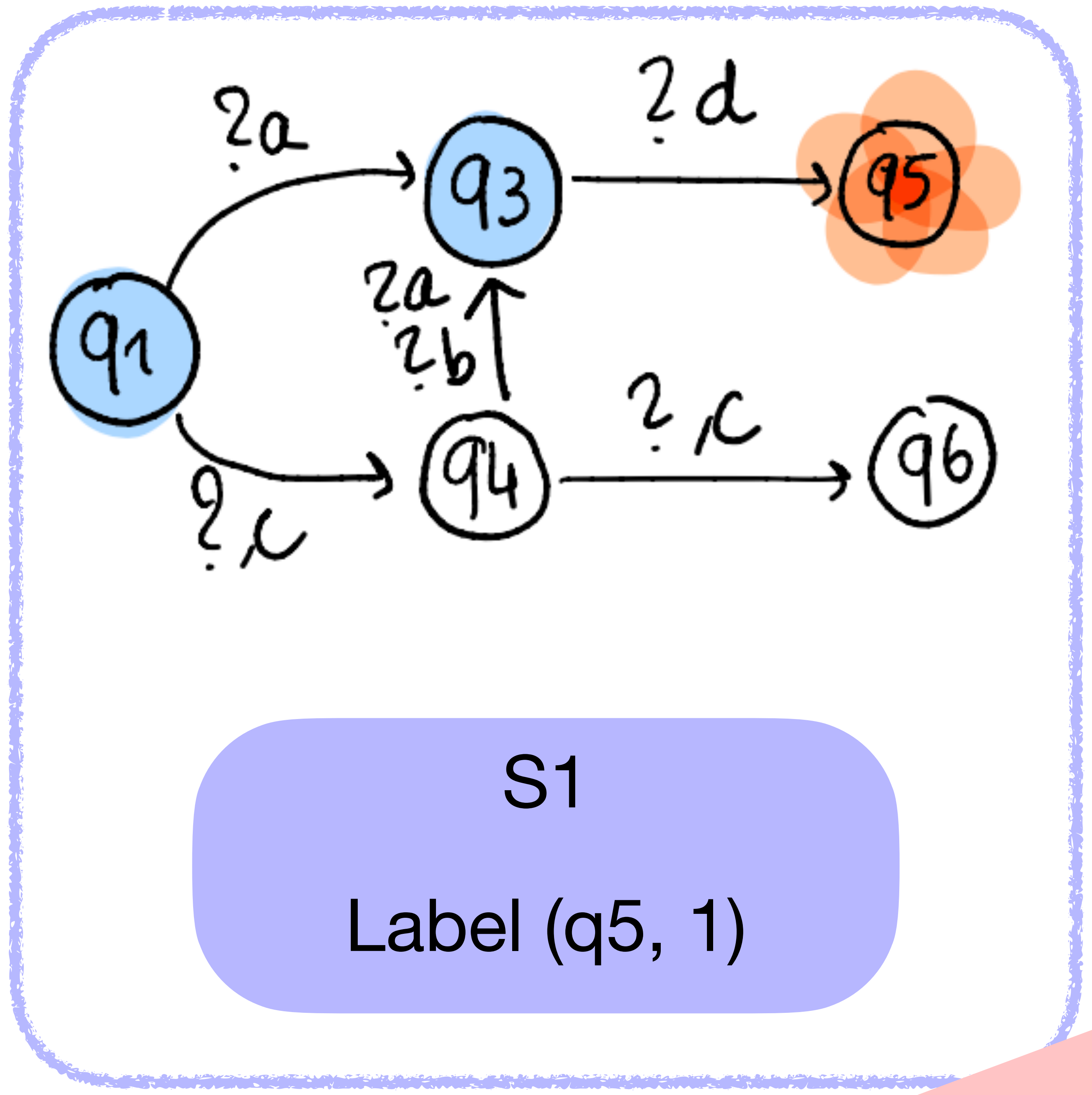
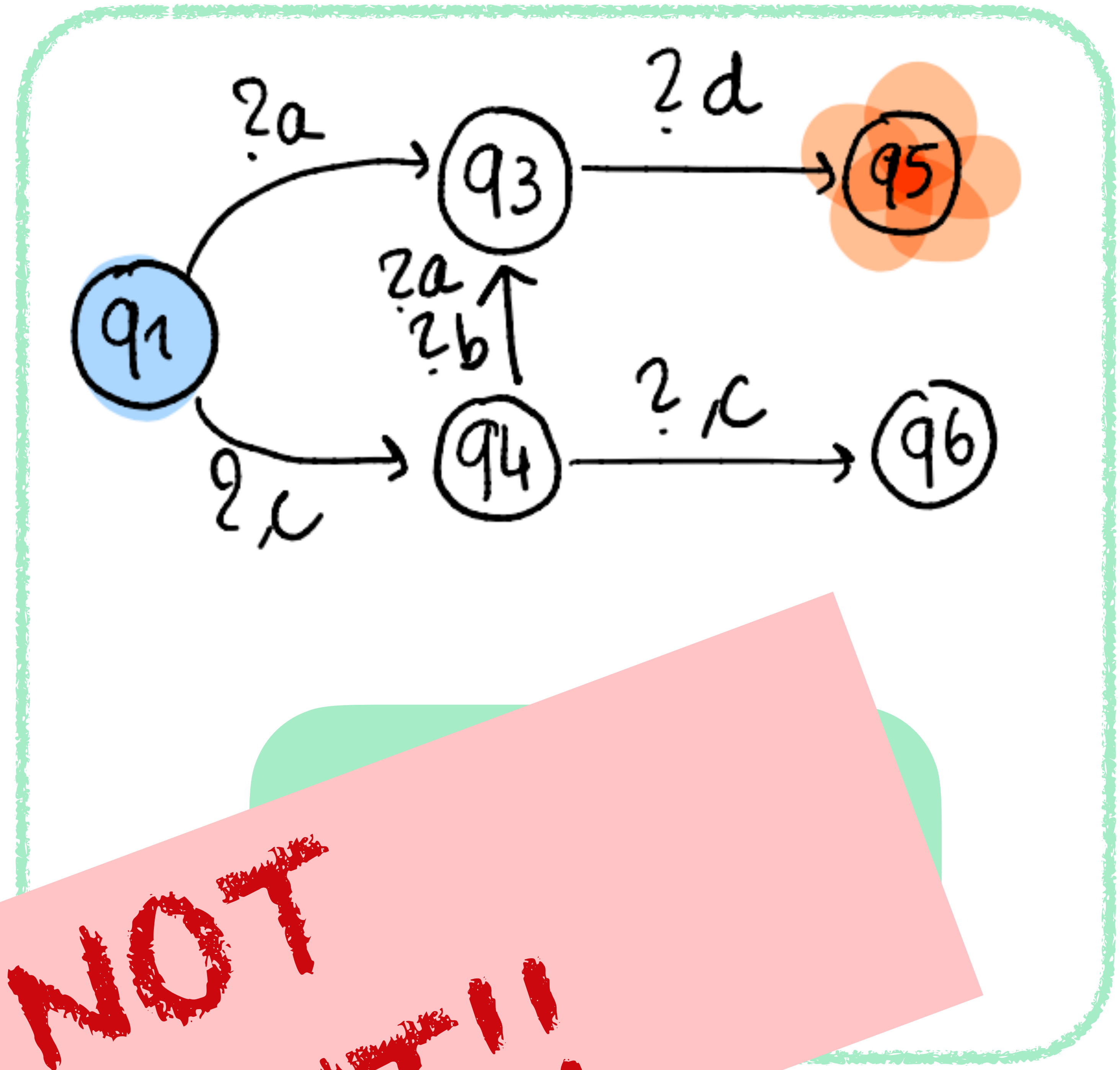
Coherent sets of Summaries



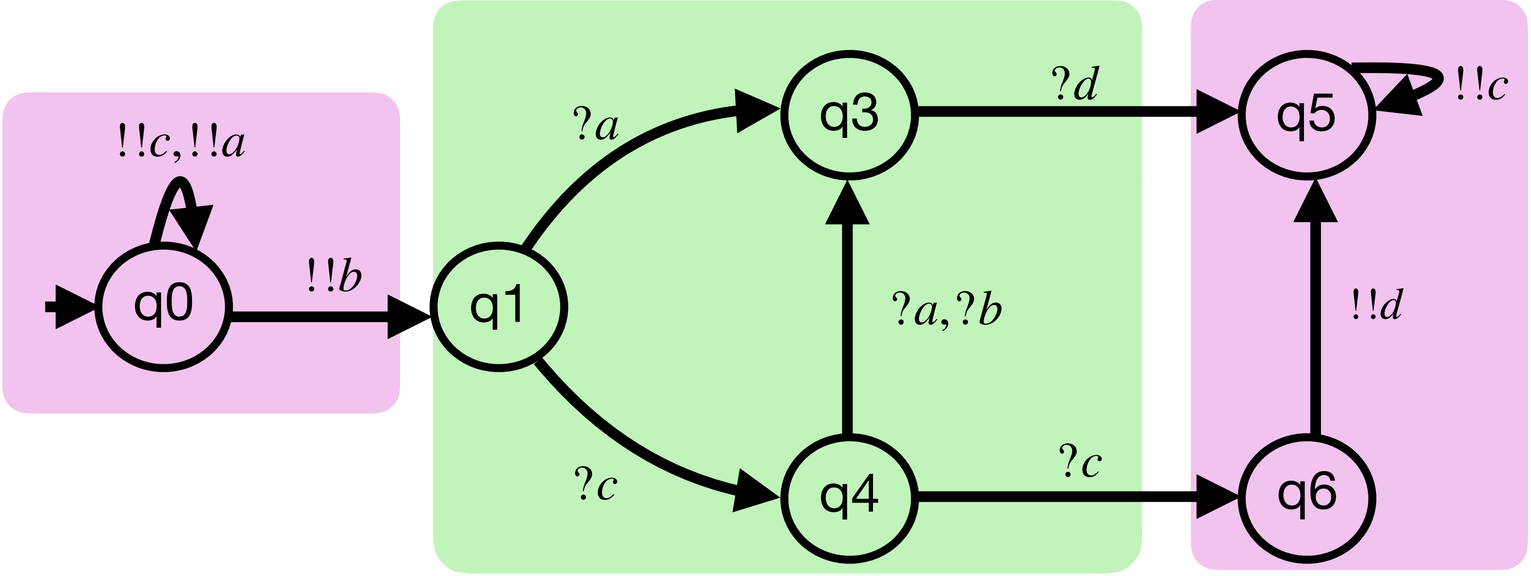


/!\ NOT COHERENT!!

* two processes on different summa… on't reach the same state OR reach the same state but not at the same time

At most #(waiting states) summaries per target states

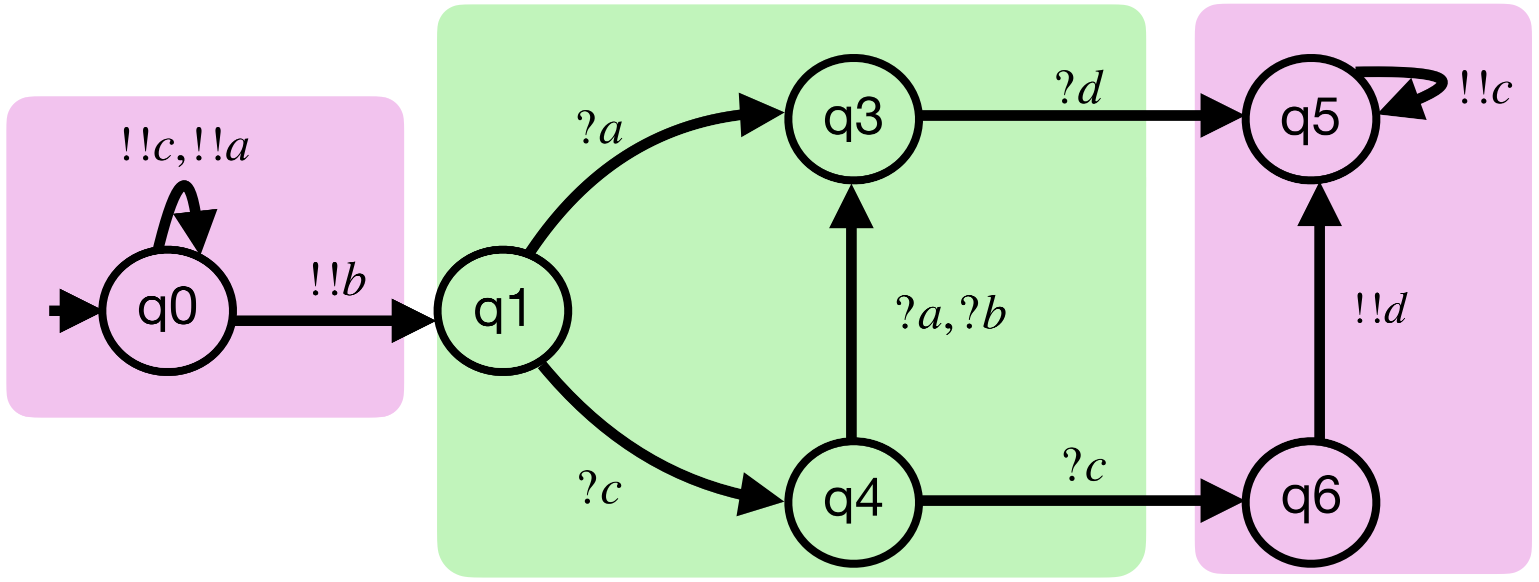
Creation of a Summary

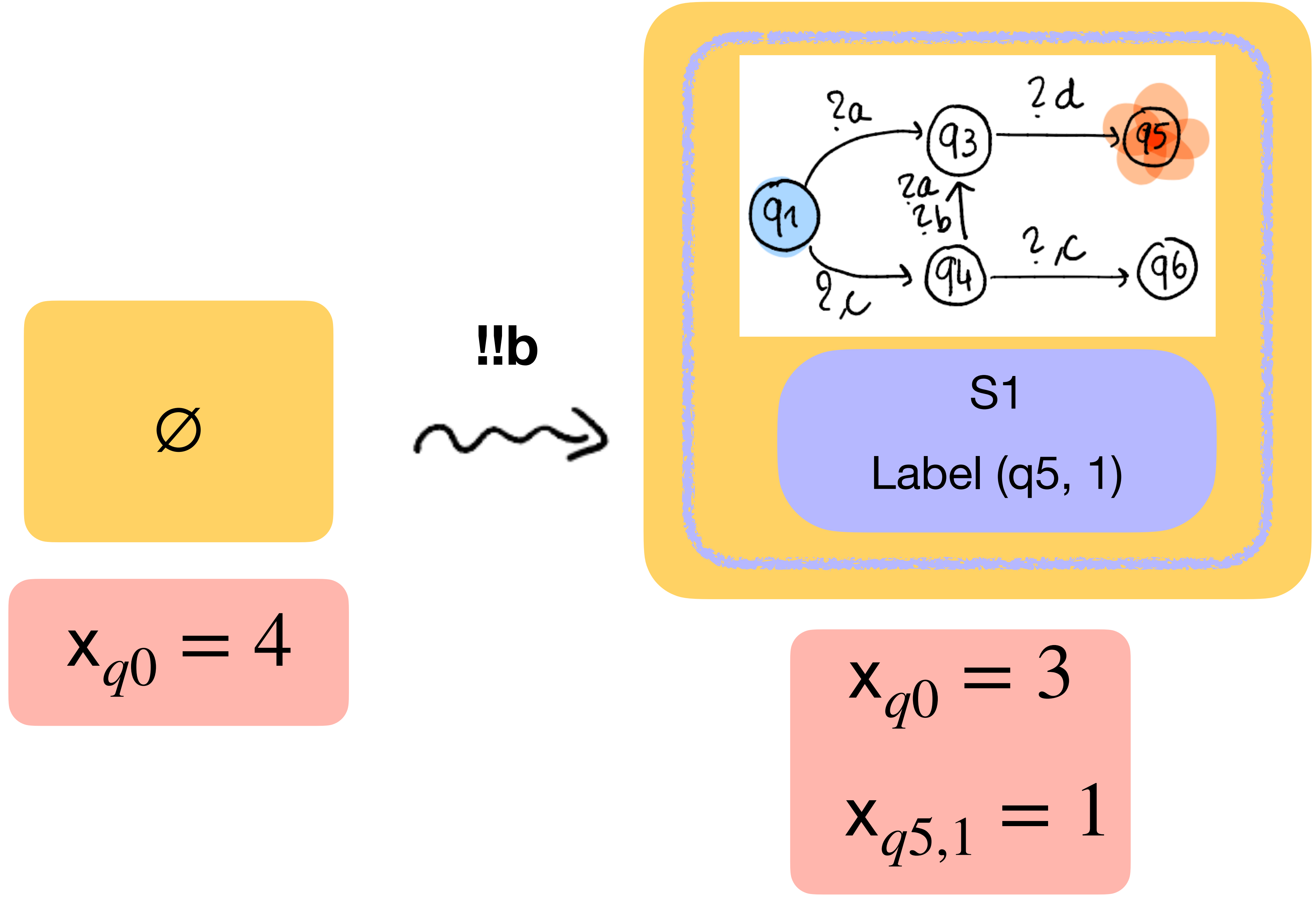


$\varnothing$

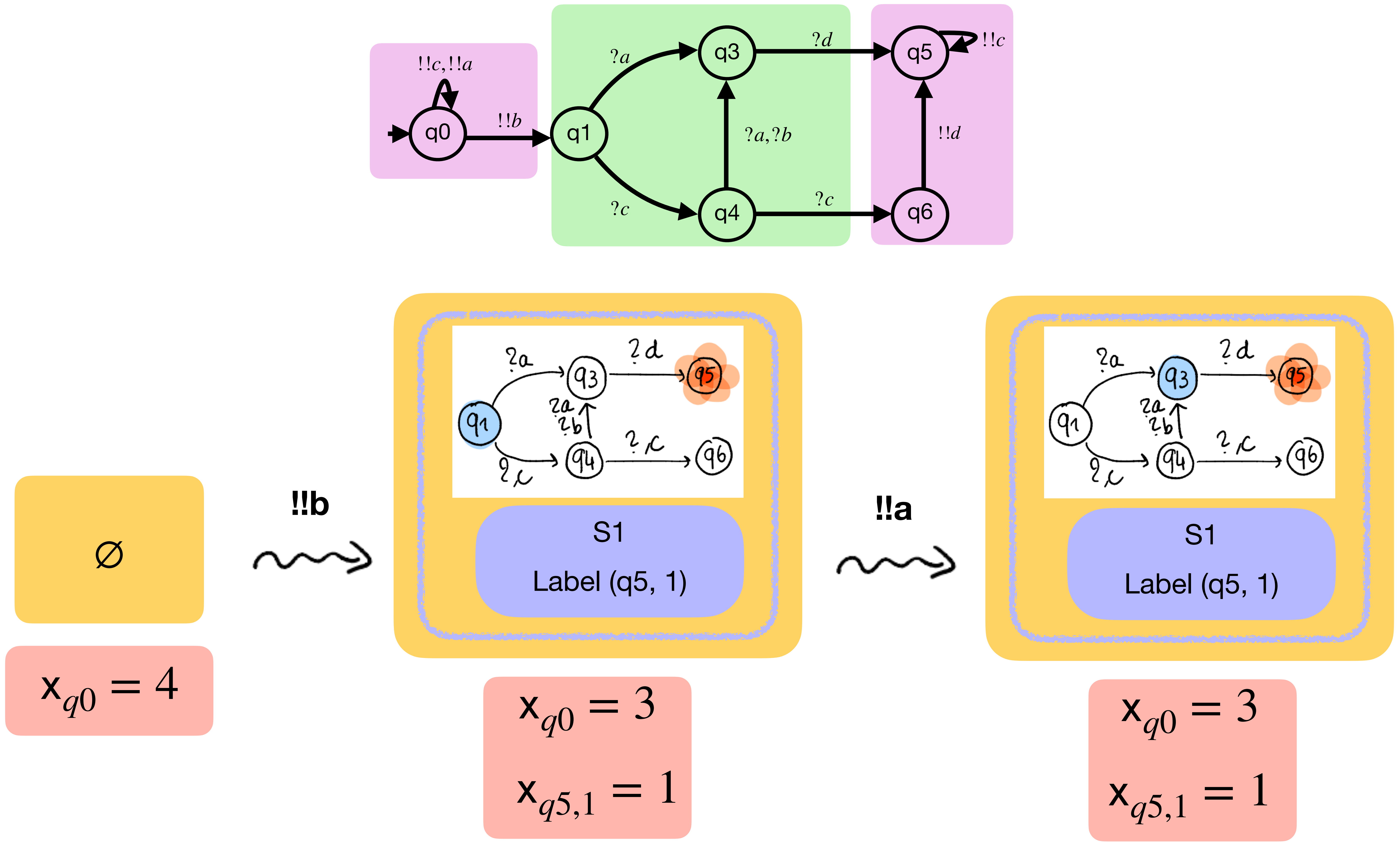
$x_{q0} = 4$

Creation of a Summary





Creation of a Summary



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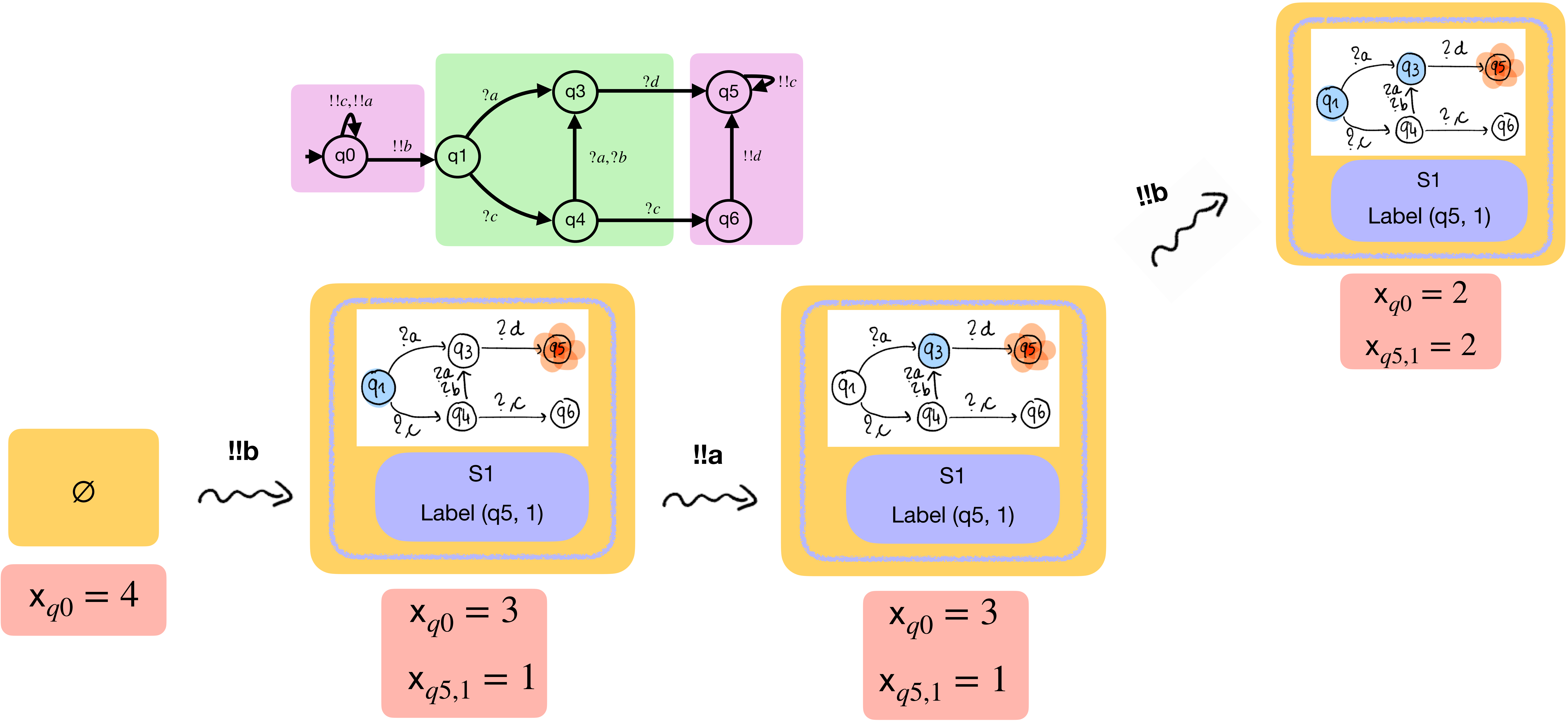
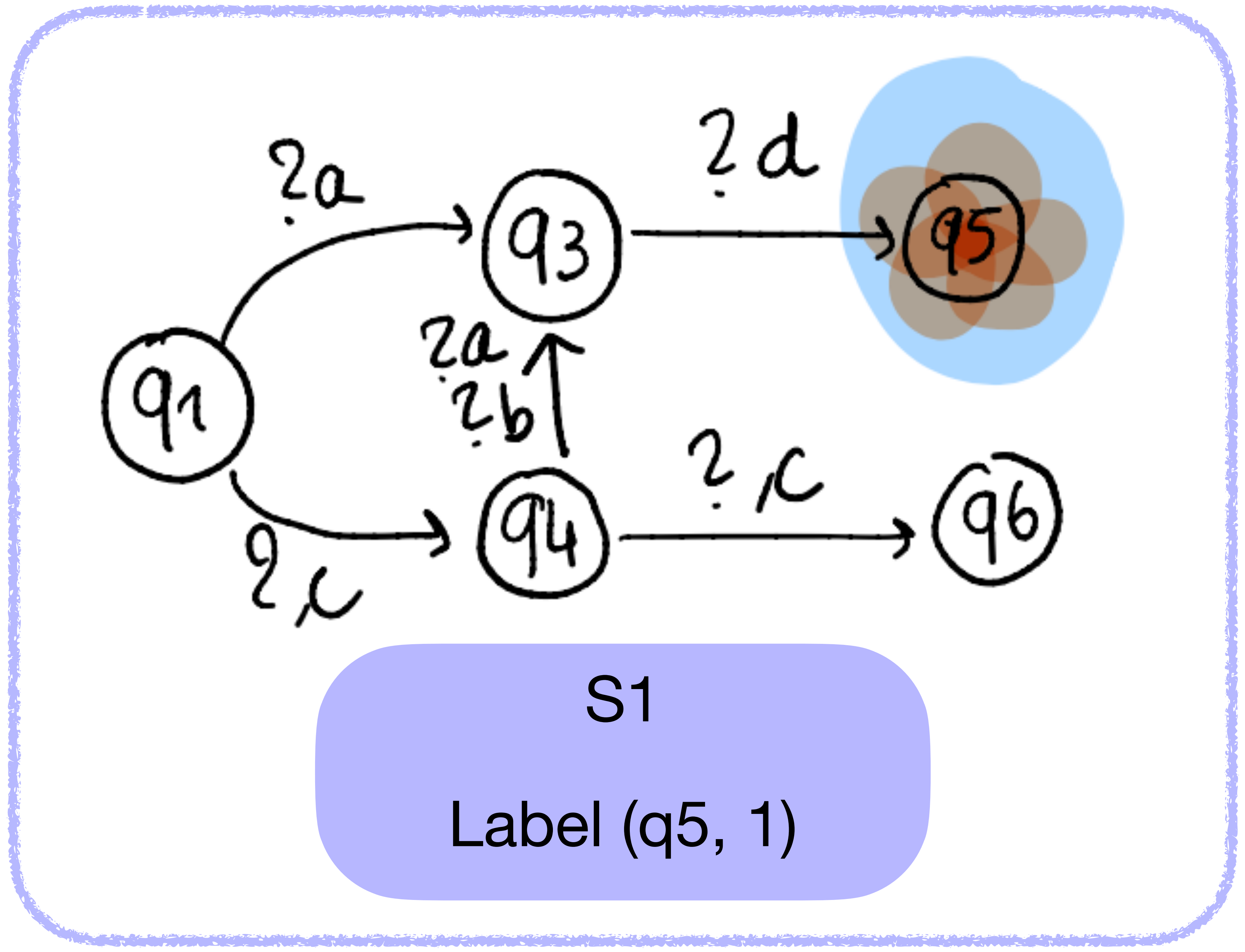


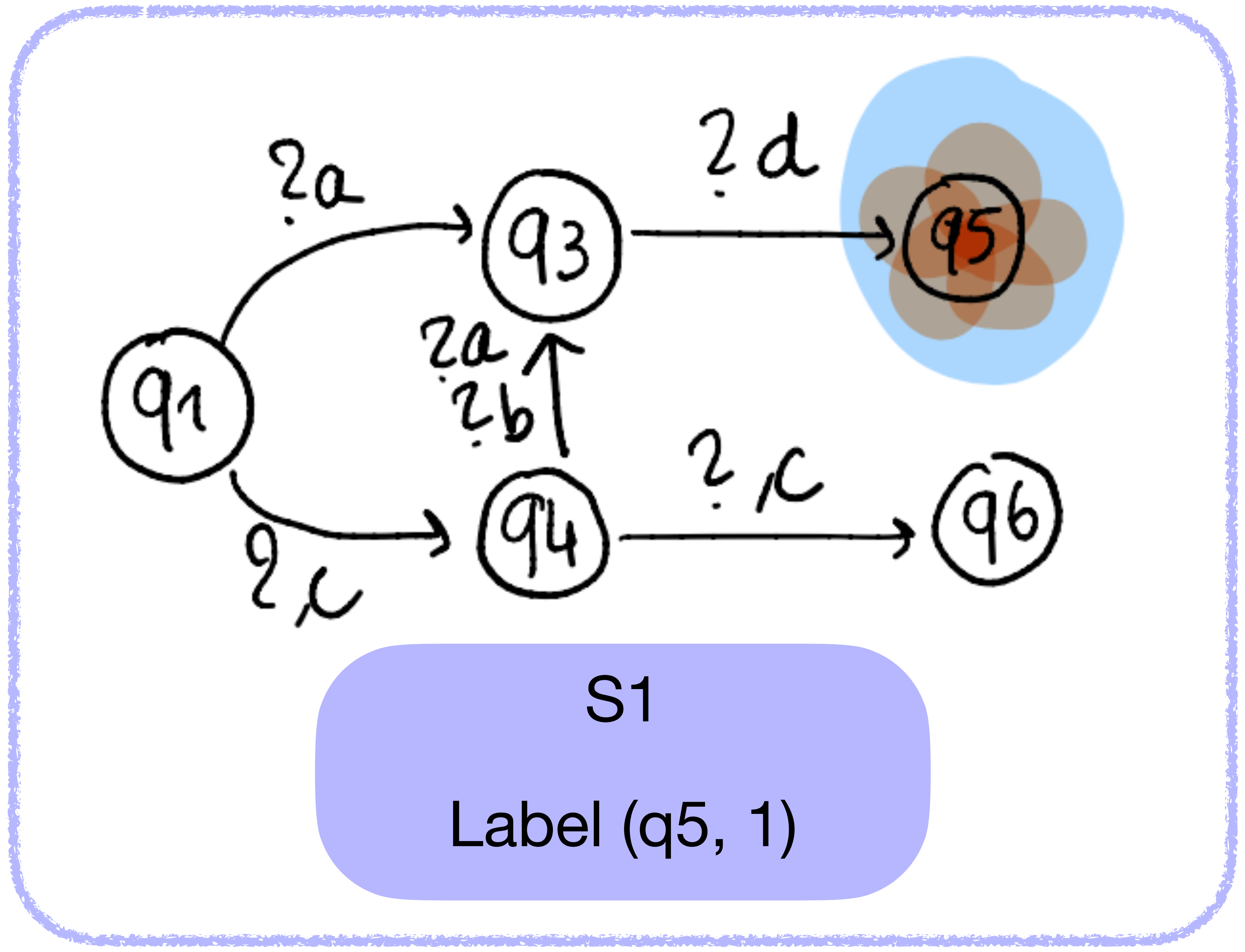
Creation of a Summary

Empty a Summary?



$x_{q5,1} = 4$

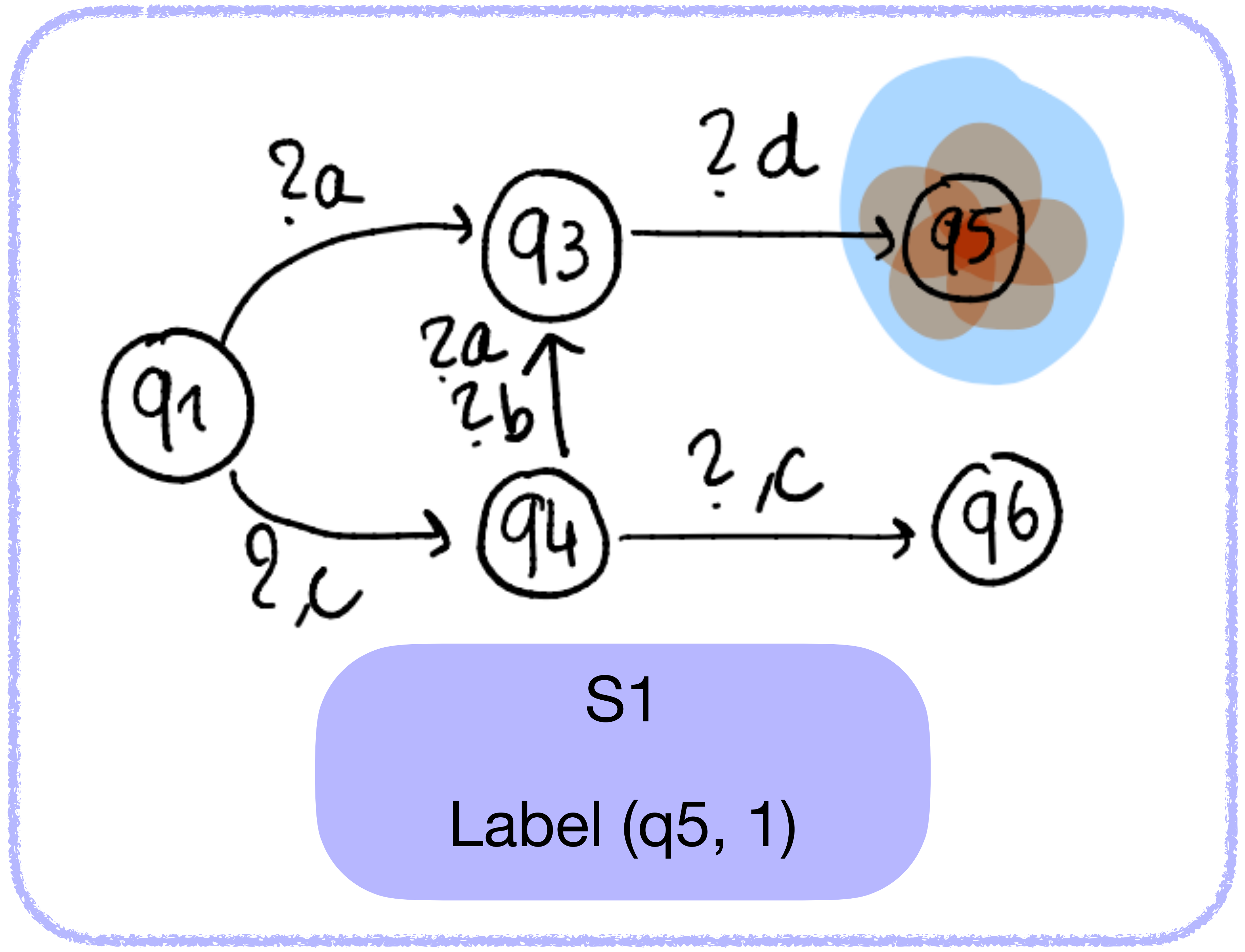
Empty a Summary?



$x_{q5,1} = 4$

Everyone has arrived on q5, what should we do?

Empty a Summary?

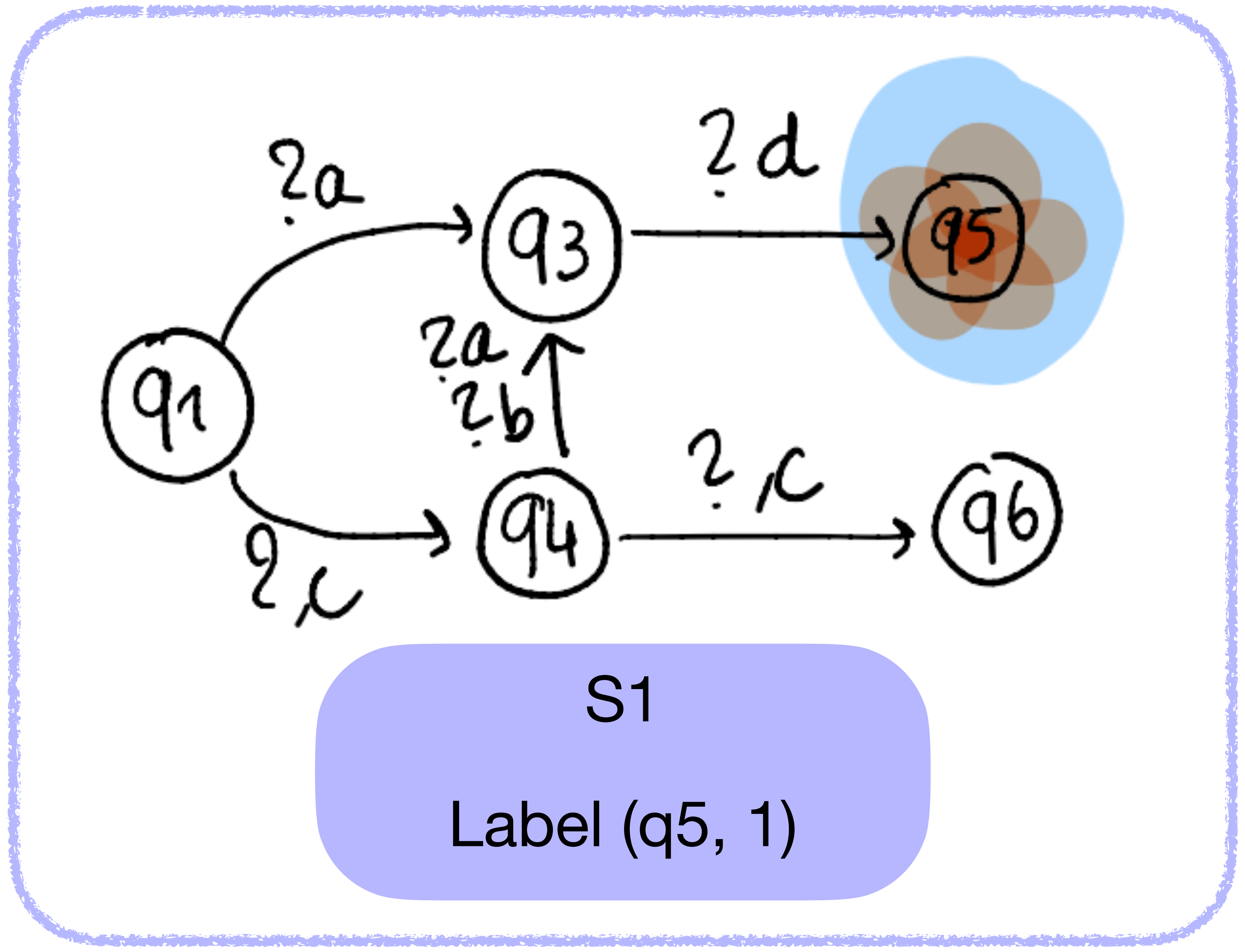


$x_{q5,1} = 4$

Everyone has arrived on q5, what should we do?

Forget about the summary

Empty a Summary?



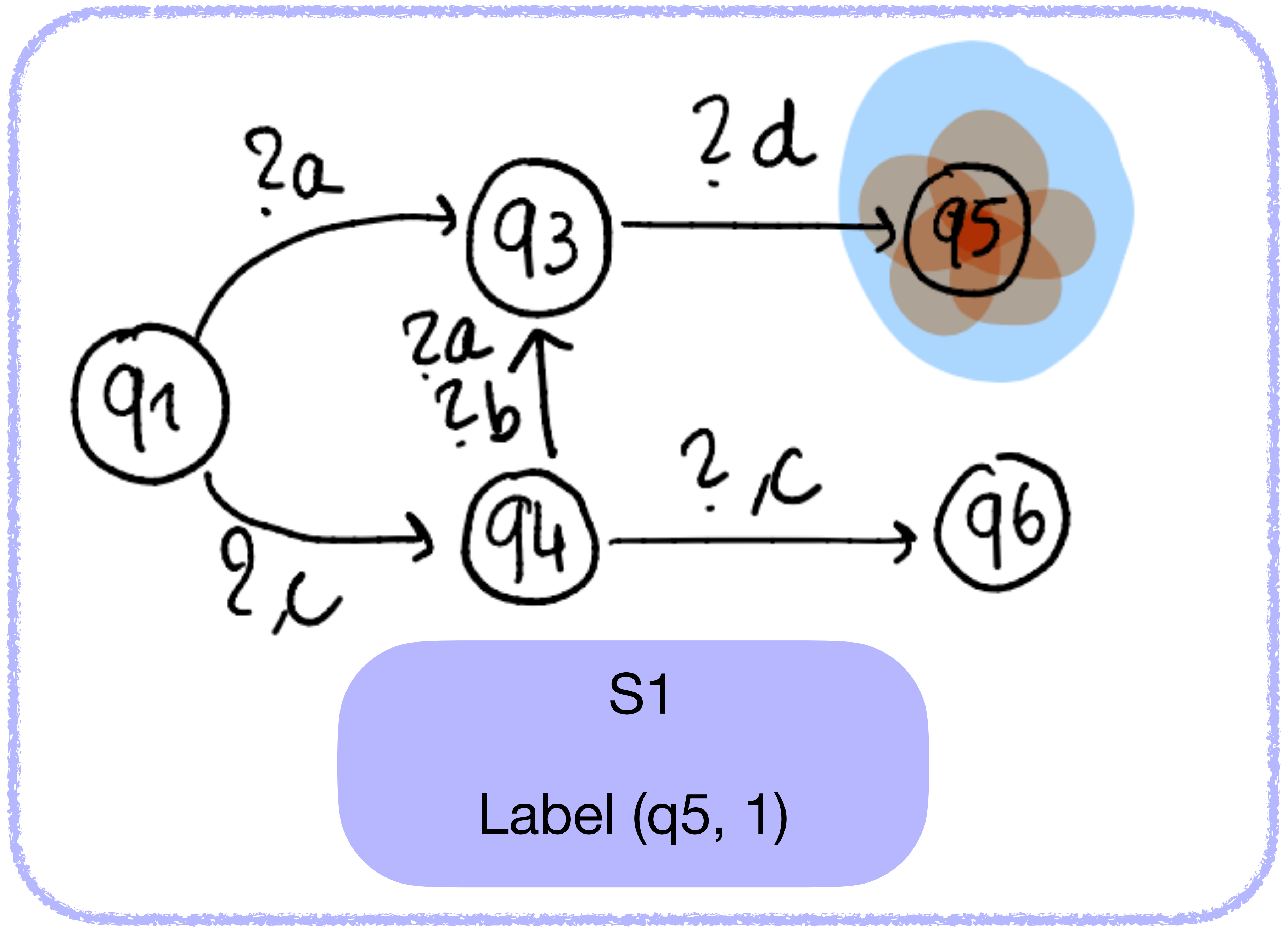
$x_{q5,1} = 4$

Everyone has arrived on q5, what should we do?

Forget about the summary

Transfer the counter $x_{q5,1}$ to x_{q5}

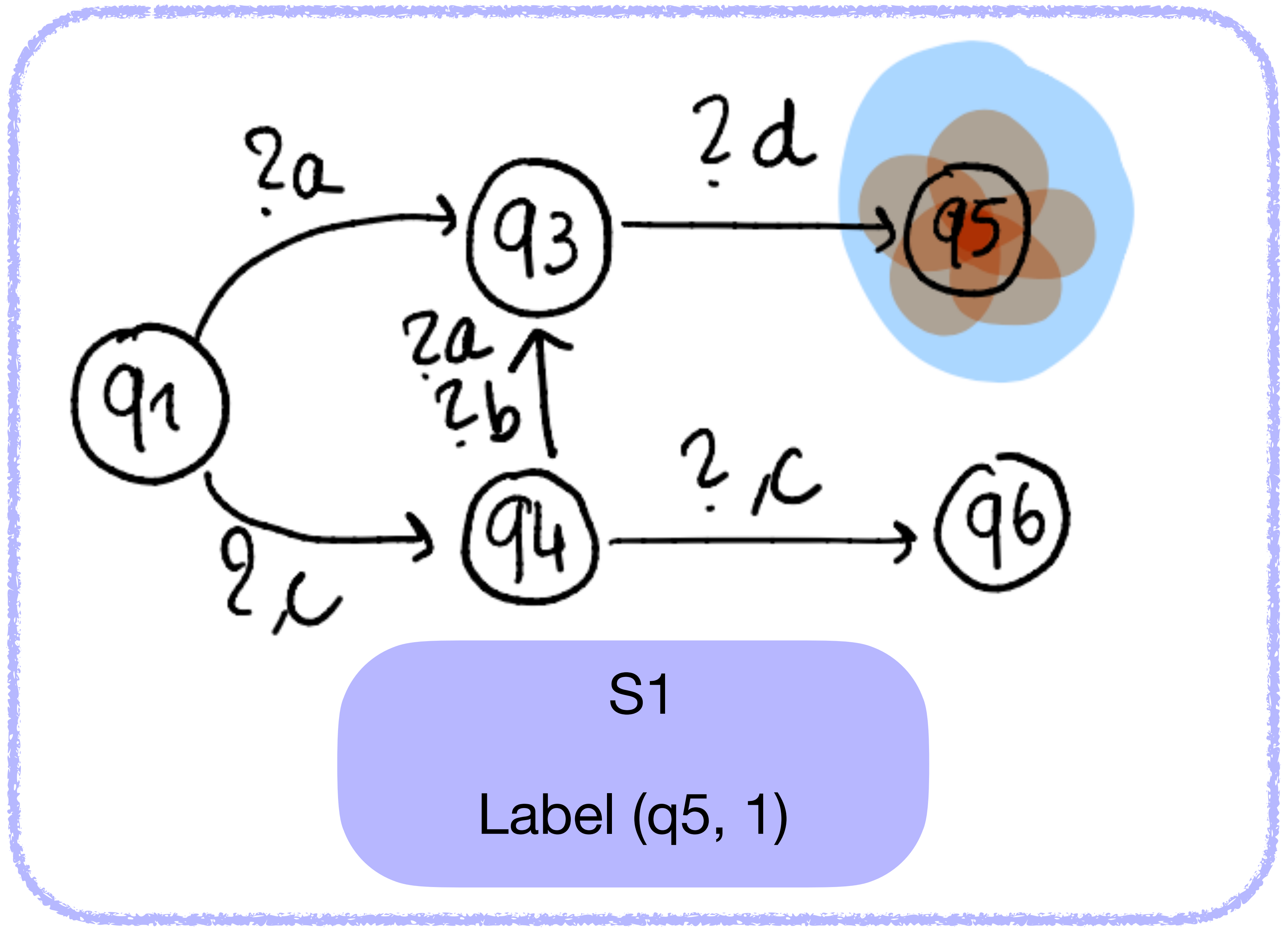
Empty a Summary?



$x_{q5,1} = 4$

$x_{q5} = 0$

Empty a Summary?



$x_{q5,1} = 4$

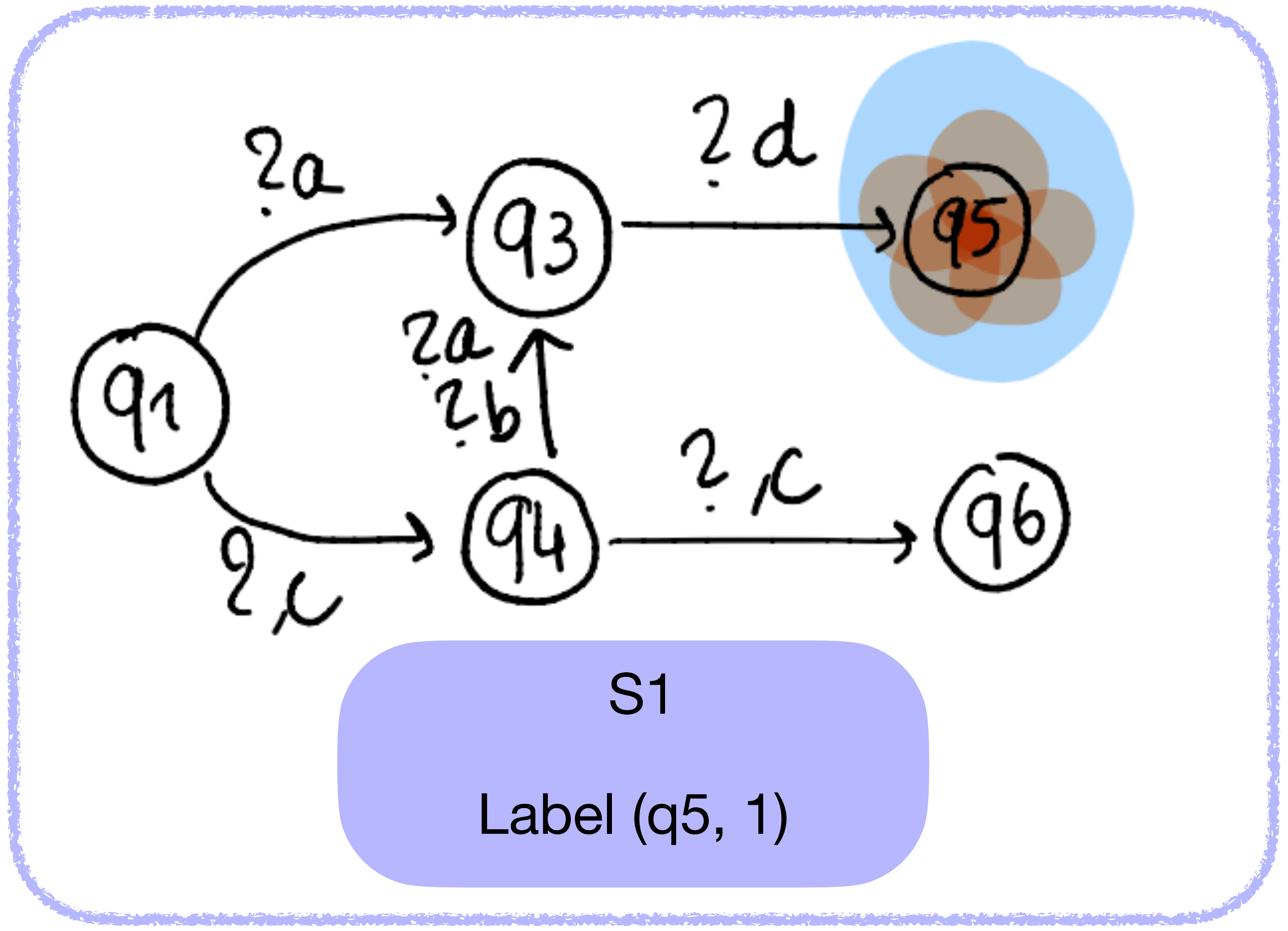
$x_{q5} = 0$

$\rightsquigarrow$

$x_{q5,1} = 0$

$x_{q5} = 4$

Empty a Summary?



$x_{q5,1} = 4$

$x_{q5} = 0$

⇝

$x_{q5,1} = 0$

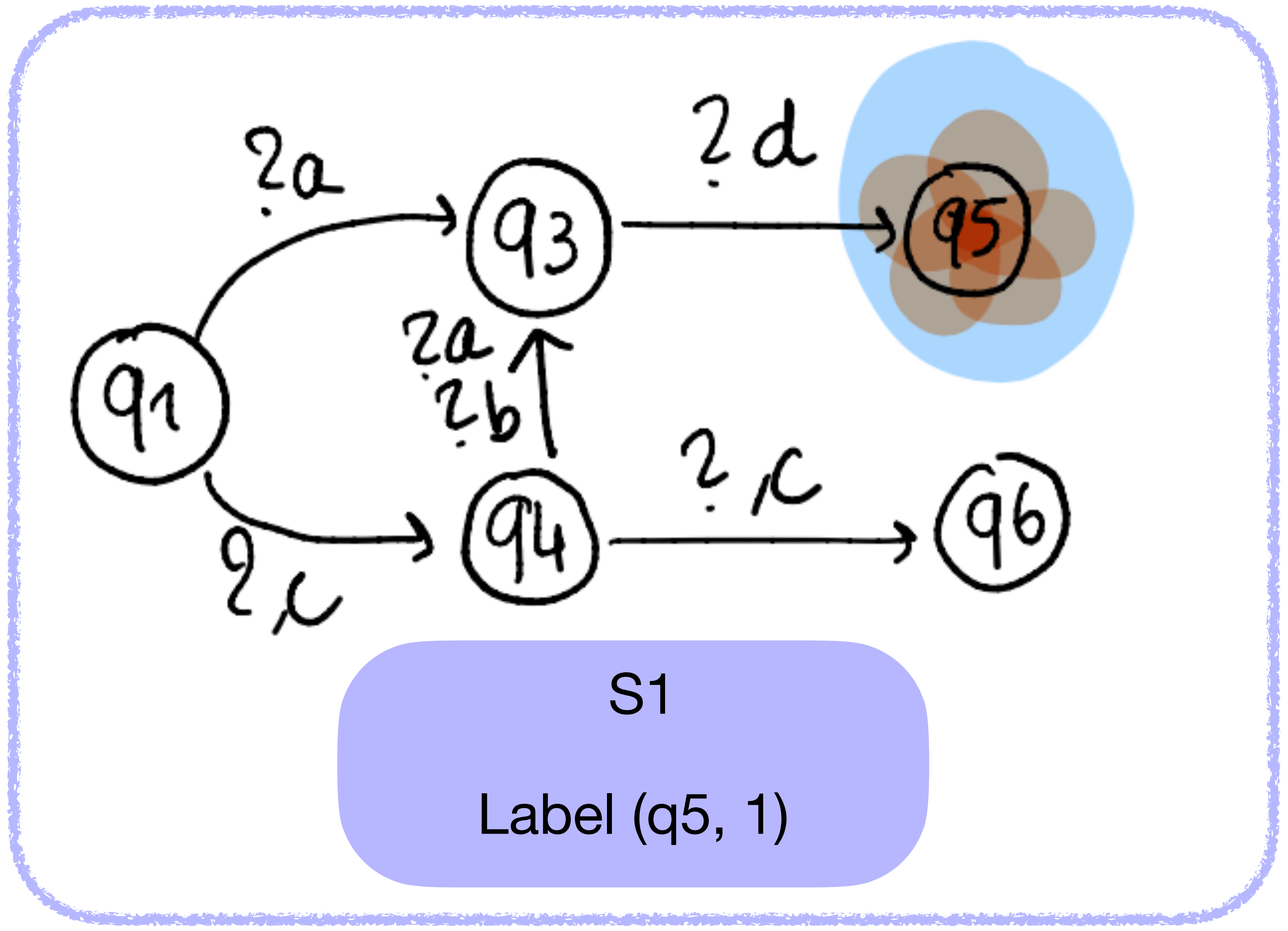
$x_{q5} = 4$

⇝

$x_{q5,1} = 1$

$x_{q5} = 3$

Empty a Summary?



$x_{q5,1} = 4$

$x_{q5} = 0$

⇝

$x_{q5,1} = 0$

$x_{q5} = 4$

⇝

$x_{q5,1} = 1$

$x_{q5} = 3$

Not a problem!

We let a process asleep on q5 until we re use label (q5,1) and re transfer the counter

Conclusion

- Reachability for Wait-Only protocols is decidable but Ackermann-hard
- Model Checking W-O protocols against LTL specification is EXPSPACE-complete (cf. [Habermehl'97])
- Single-Wait-Only protocols

Thank you!