

# Verification of dynamic systems with locks and variables

Corto Mascle

joint work with Anca Muscholl, Igor Walukiewicz

*PaVeDys*

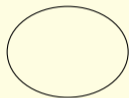
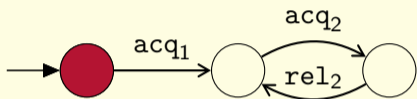
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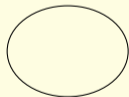
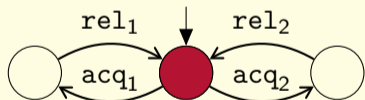
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# Lock-sharing systems<sup>1</sup>



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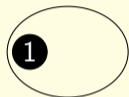
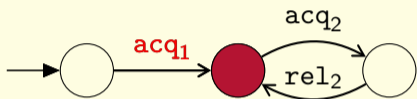


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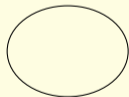
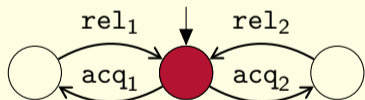
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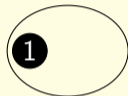
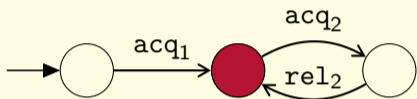


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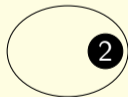
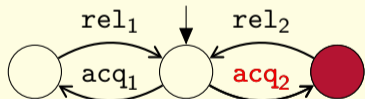
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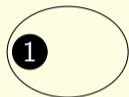
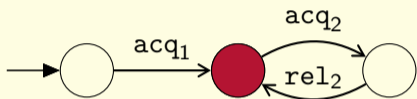
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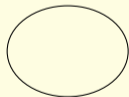
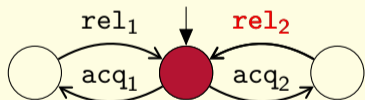
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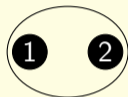
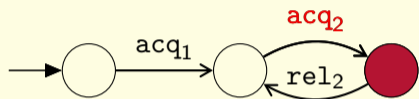
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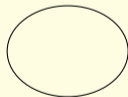
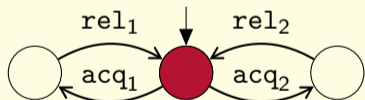
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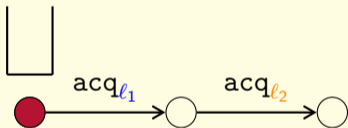
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## Restriction: Nested locking

All processes acquire and release locks in a **stack-like order**, i.e., a process can only release the lock it acquired the latest.

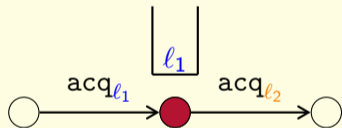


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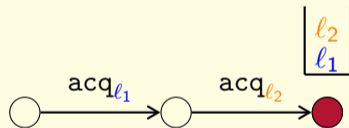
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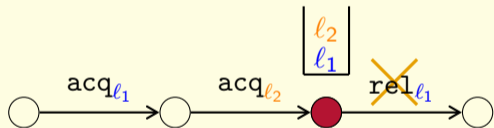
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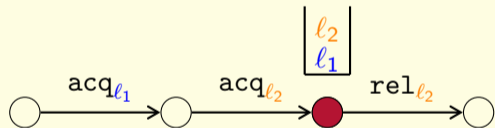
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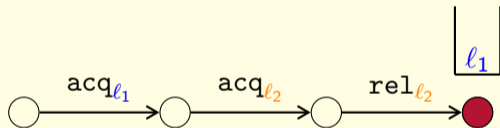
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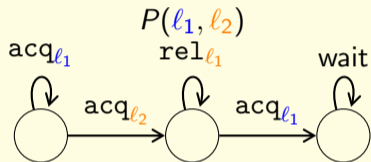
- ▷ We want to allow an unbounded number of processes and locks.
- ▷ A process can spawn other processes
- ▷ A process takes parameters, represented by *lock variables*

$$Proc = \{P(l_1, l_2), Q(l_1, l_2, l_3), R(), \dots\}$$



# Dynamic LSS<sup>2</sup>

Locks : ■ ■

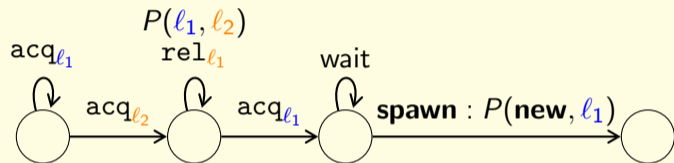


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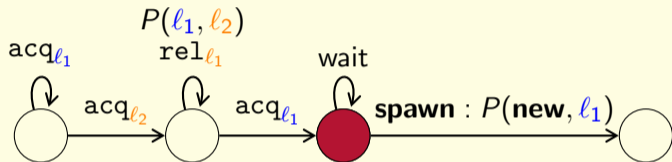
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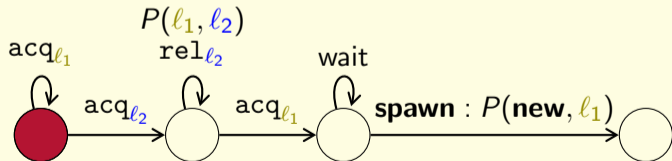
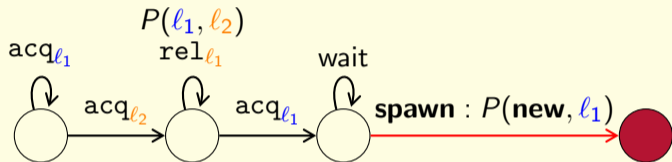
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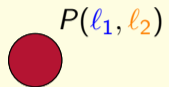
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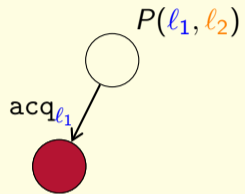


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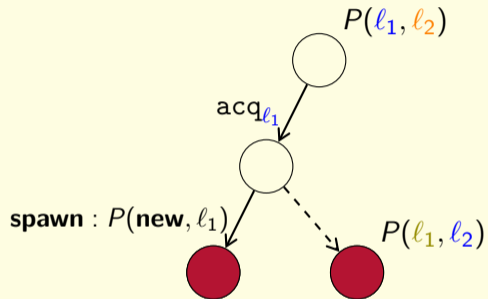
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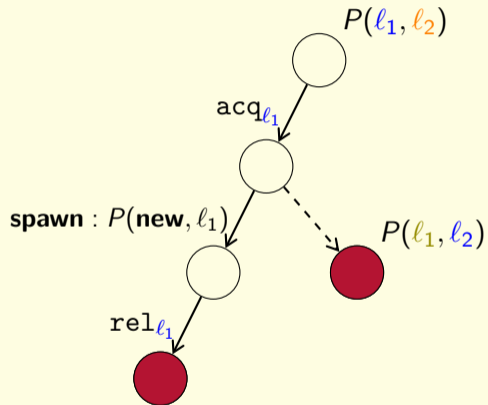
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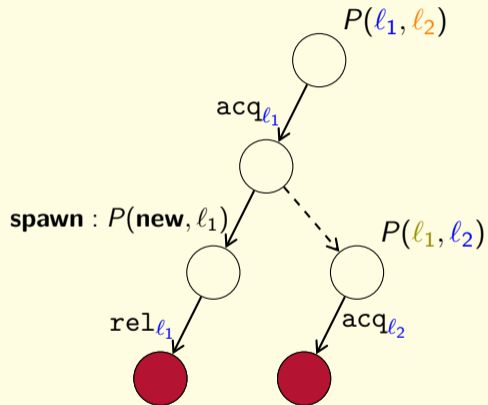


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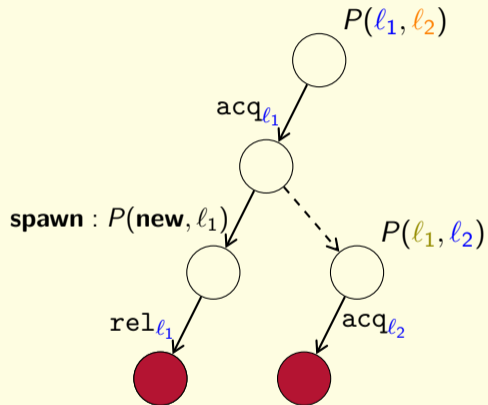




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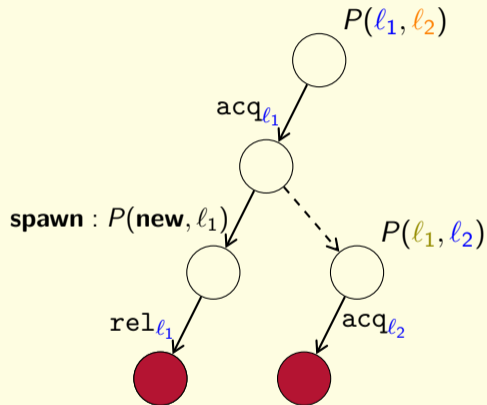


## Tree representation



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*“Every process is blocked after some point”*

*“Finitely many processes are spawned”*

*“Infinitely many processes reach an error state  $q_{err}$ ”*

Deadlocks

## Regular model-checking problem

**Input:** A DLSS  $\mathcal{D}$  and a parity tree automaton  $\mathcal{A}$ .

**Output:** Is there a run of  $\mathcal{D}$  accepted by  $\mathcal{A}$ ?

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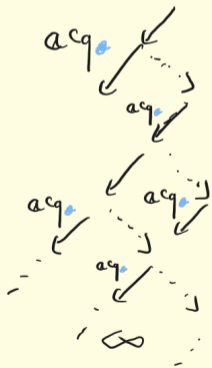
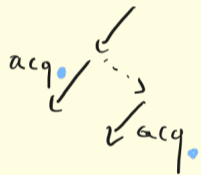
**Problem:** characterise trees that represent actual executions.

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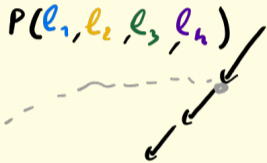


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For each node we guess a label of the form

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- ▶  $l_3 < l_1$   
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$P(l_1, l_2, l_3, l_4)$



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The automaton checks that:

- ▶ the labels are consistent
- ▶ There exists a well-founded linear ordering on locks in which all local orders embed. (Technical part, also see related work [Demri Quaas, Concur '23])

Theorem [M., Muscholl, Walukiewicz Concur 2023]

Regular model-checking of DLSS is EXPTIME-complete, and PTIME for fixed number of locks per process and parity index.

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**What about pushdown processes?**

## Right-resetting pushdown tree automata

**Right-resetting** = the stack is emptied every time we go to a right child.

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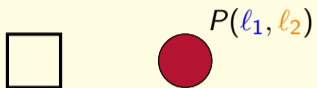
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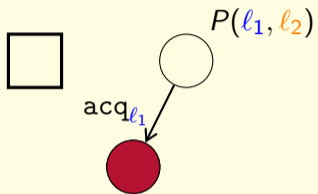
**What about shared variables?**

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We add a register and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

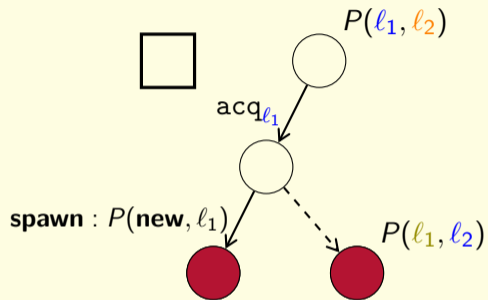
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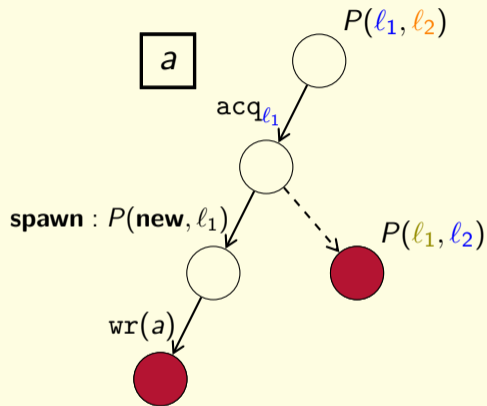


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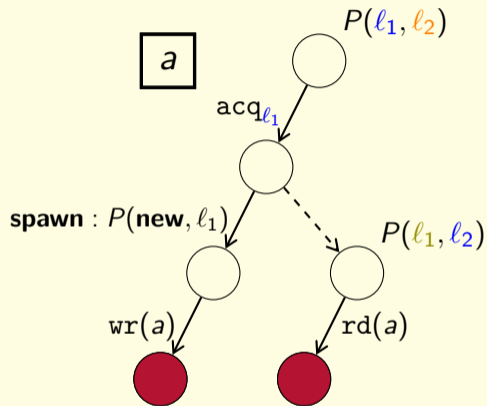
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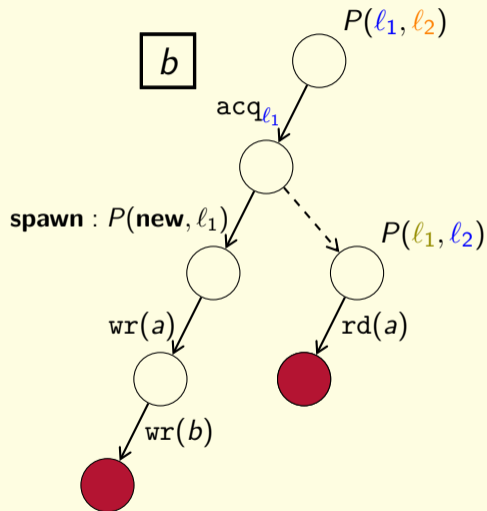
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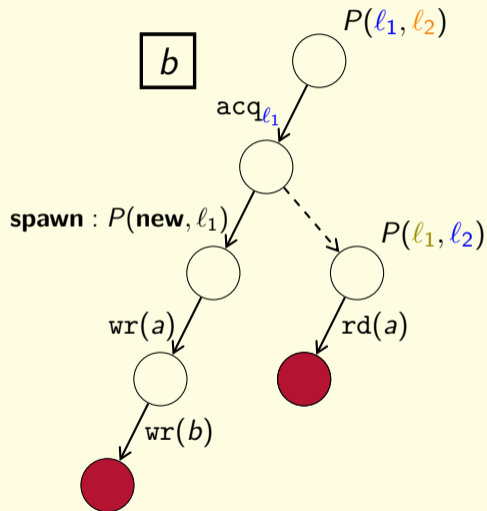
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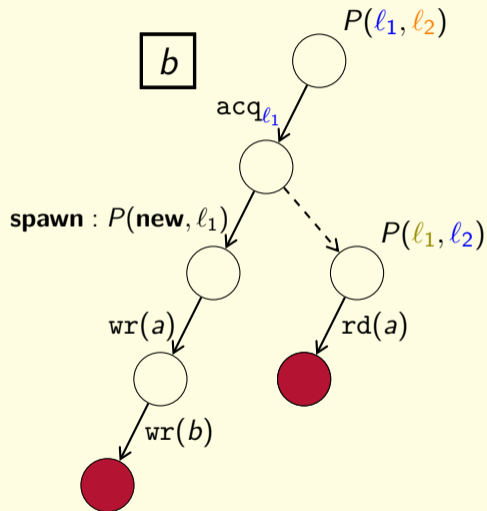
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State reachability is undecidable for DLSS with variables.

## Bounded writer reversals

Writer reversal = the process writing in the shared register changes.

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Every finite run with a single writer can be cut into  $2^{O(|Q|)}$  phases.

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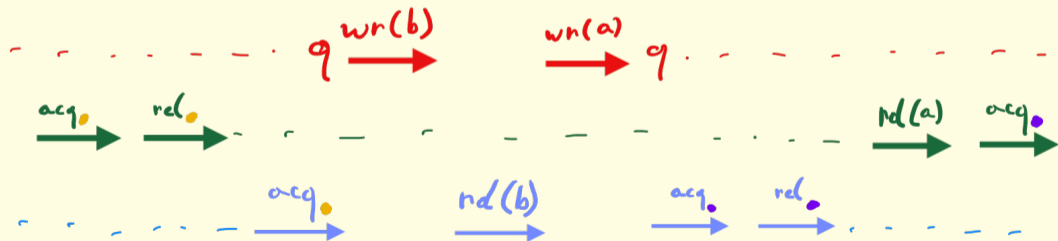
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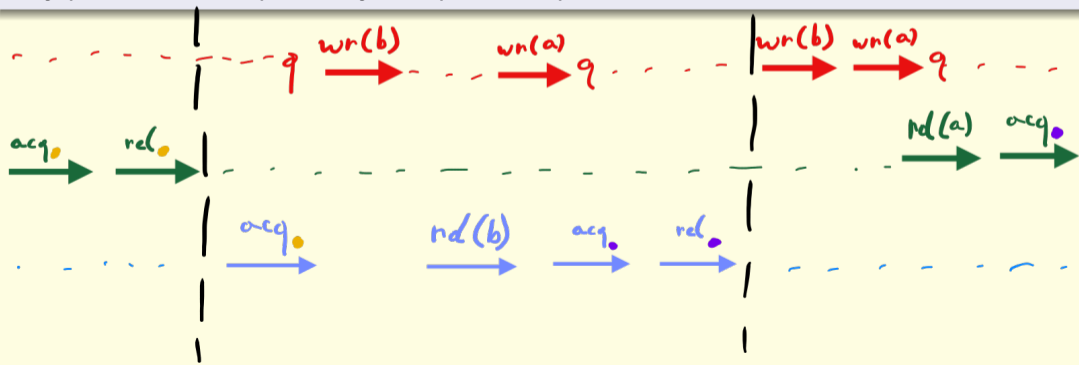


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*Thanks!*