On the Send-synchronizability problem for Mailbox Communication

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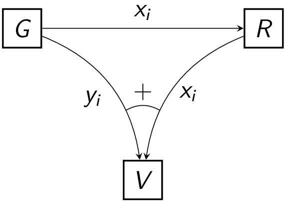

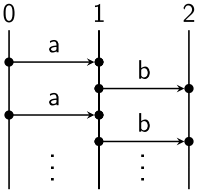

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2 Send-synchronizability

3 1-schedulability

4 Fully-matched & Good traces

5 1-sched & Bad traces

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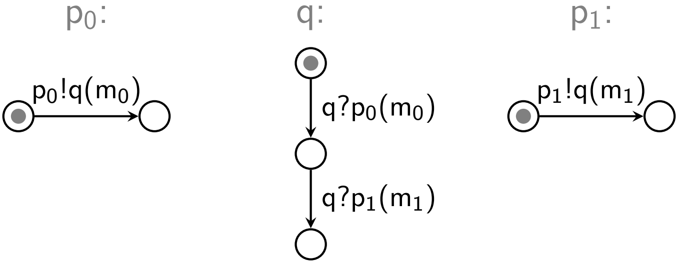
CFM: Communicating Finite-state Machines (set of processes sending/receiving messages).



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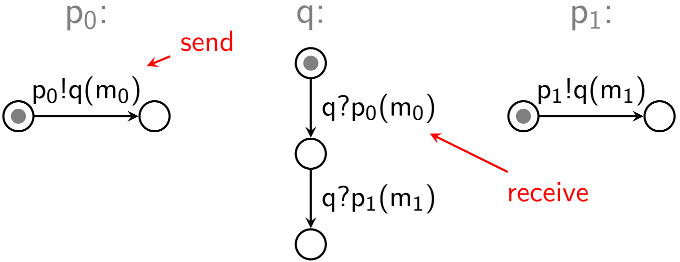

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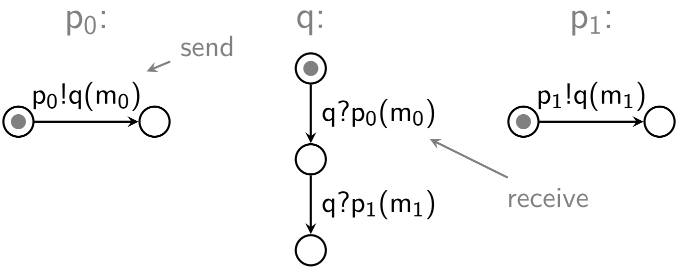

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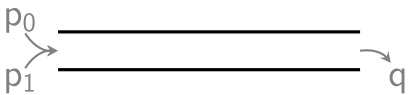

One *FIFO* channel per process.

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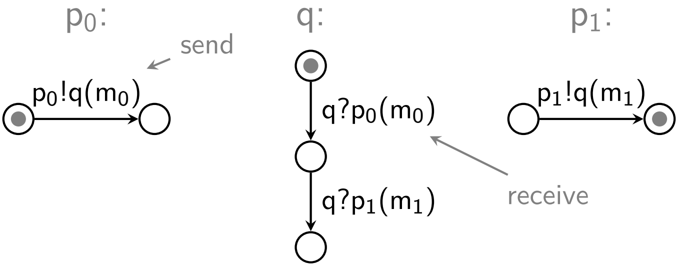

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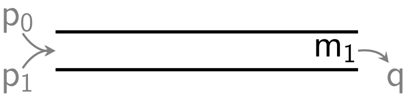

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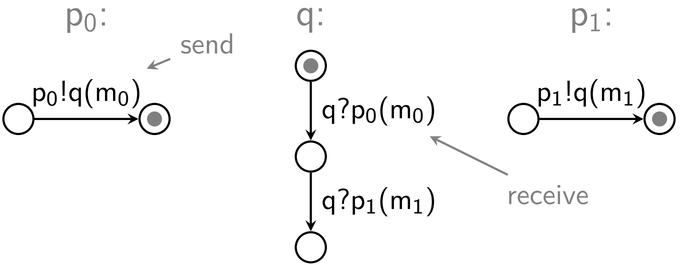

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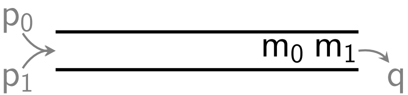

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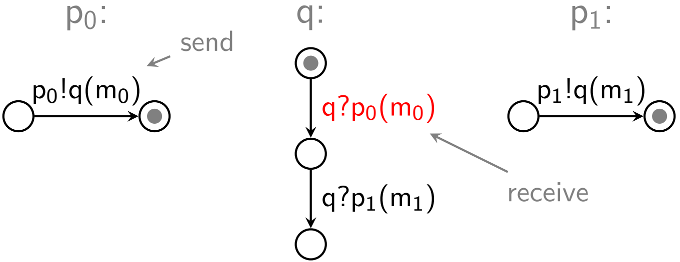

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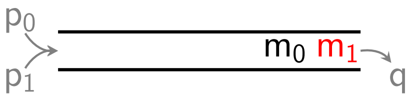

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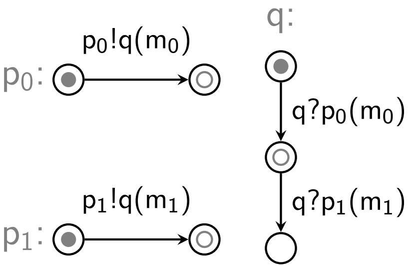
Mailbox executions

Executions that are possible for peer-to-peer communication may not be possible for mailbox.

Message Sequence Charts

Multiple executions with same effect on system.



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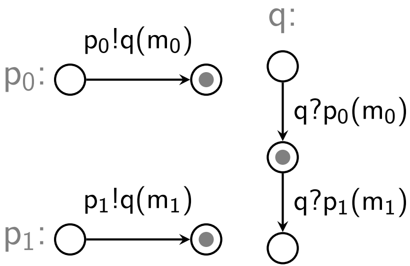

exec 0:	exec 1:
$p_0!q(m_0)$	$p_0!q(m_0)$
$q?p_0(m_0)$	$p_1!q(m_1)$
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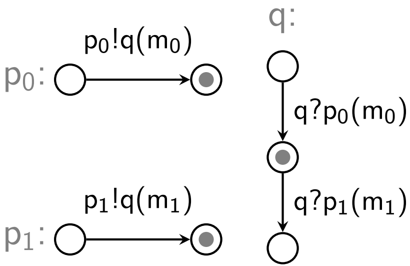

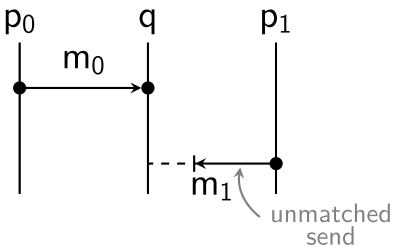

Message Sequence Charts (MSC)

Partial-order representation of behaviors of a CFM (order between events on a same process, between paired send/receive & *mailbox order*).

Two equivalent executions have the same MSC.

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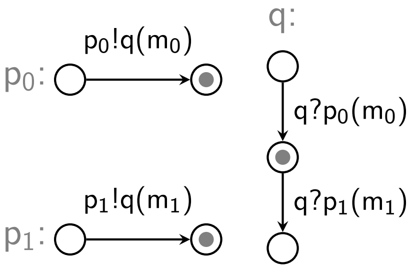

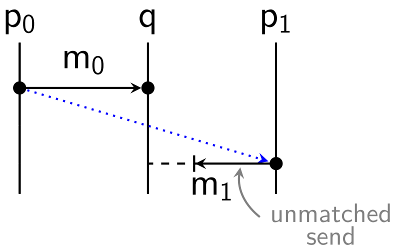

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Mailbox order: two sends to the same process are ordered if the first one is matched.

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Send-synchronizability

$$w = p_0!q(m_0)\, q?p_0(m_0)\, p_1!q(m_1)$$

projection on sends of w

$$w|_S = p_0!q(m_0) \qquad\qquad p_1!q(m_1)$$

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Executions

For a CFM $\mathcal{A}$.

- $Tr(\mathcal{A})$ set of all executions of $\mathcal{A}$.
- $Tr_{\mathtt{rdv}}(\mathcal{A})$ set of all executions where sends are directly followed by their matching receive.

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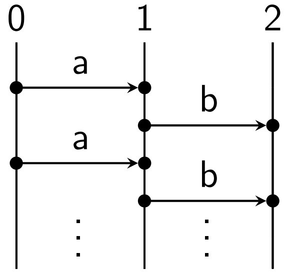

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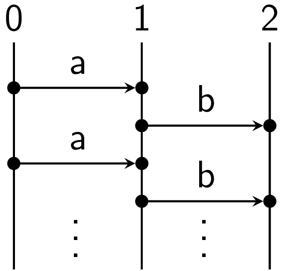

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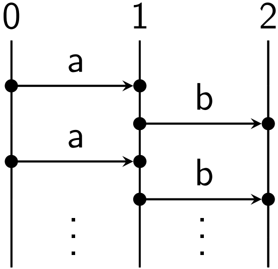

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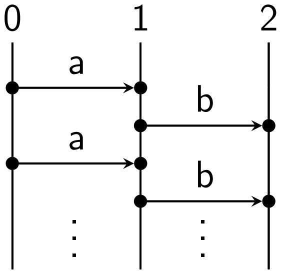

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Send-Synchronizability

A CFM $\mathcal{A}$ is called *sendsend-synchronizable* if
$Tr(\mathcal{A})|_S = Tr_{\mathtt{rdv}}(\mathcal{A})|_S$.

History of Send-synchronizability

- [Basu & Bultan, 2012/2016] Send-synchronizability is decidable for mailbox and peer-to-peer CFMs.

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Is Send-synchronizability decidable for mailbox CFMs?

If not, is there a subclass of CFMs such that Send-synchronizability is decidable ?

Undecidability (short)

PCP (variant)

A set of pairs $(x_1, y_1), \ldots, (x_K, y_K)$ of words over an alphabet Σ.
Is there a sequence of indices $i_1, \ldots, i_n \in \{1, \ldots, K\}$ such that

- $i_1 = 1$
- $x_{i_1} \ldots x_{i_n} = y_{i_1} \ldots y_{i_n}$
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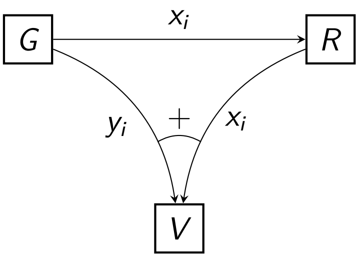

- **G**uess guesses the sequence of indices, sends x_i to **R**elay and y_i to **V**erif.
- **R**elay either delays letters of x_i to intertwine them with letters of y_i to **V**erif, or can receive and send anything (dummy).
- **V**erif checks that the two words are identical.

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Restriction: 1-Schedulable

1-schedulability

A trace is a 1-*scheduling* if every send is either followed by its receive, or is unmatched.

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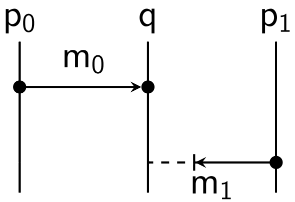

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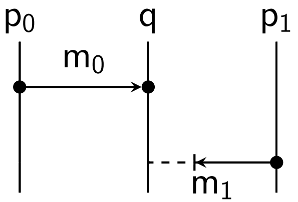

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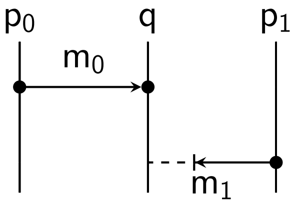

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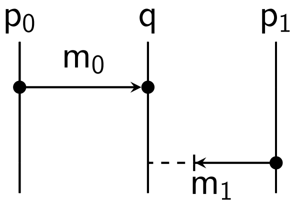
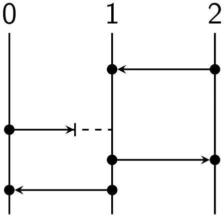


Fully-matched 1-schedulings of a CFM = rendez-vous traces.

Checking 1-schedulability

- $<_{\mathbb{P}}$ order between event on same process.
- msg order between a send and its receive.
- $<_{\mathtt{mb}}$ mailbox order.



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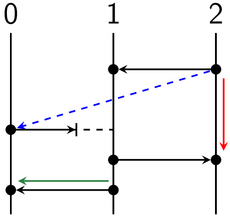

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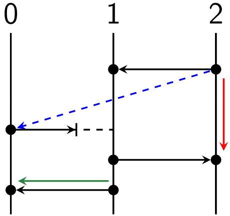

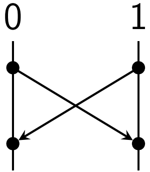

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there is a non-trivial $(<_{\mathbb{P}} \cup <_{\texttt{mb}} \cup \mathrm{msg} \cup \mathrm{msg}^{-1})$-cycle.

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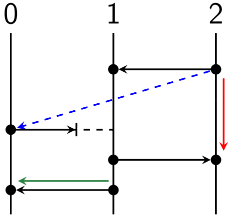

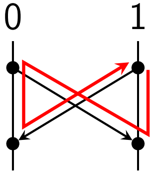

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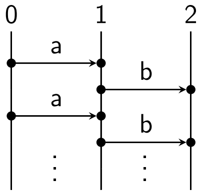
1-schedulability [Delpy, Muscholl & Sutre 2024]

The question whether a CFM is 1-schedulable is PSPACE-complete.
The language of 1-scheduling of a CFM is regular.

Send-sync & 1-sched

Even for 1-schedulable CFMs, send-synchronizability is still hard:



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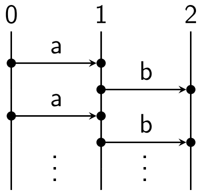

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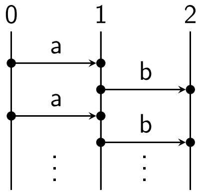

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Two cases:

- Restriction of $Tr(\mathcal{A})$ to fully matched traces.
- Include not fully matched traces.

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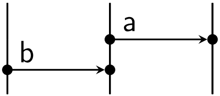
Fully-matched traces



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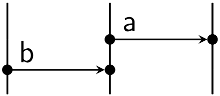

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Commutation: $SI \subseteq S \times S$. Let $a = p!q(m)$ and $b = p'!q'(m')$, $(a, b) \in SI$ if $p \neq p'$, $q \neq q'$ and $q \neq p'$

If $(a, b) \in SI$, $u, v \in S^*$: $uabv \Rightarrow_{SI} ubav$.

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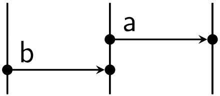

Commutation: $SI \subseteq S \times S$. Let $a = p!q(m)$ and $b = p'!q'(m')$, $(a, b) \in SI$ if $p \neq p'$, $q \neq q'$ and $q \neq p'$

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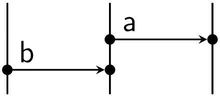

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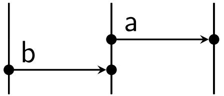

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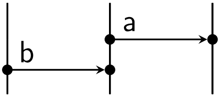

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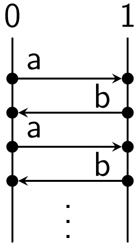
One can check if $\mathcal{L} \subseteq S^*$ regular is closed under SI.

Send-sync for fully-matched 1-schedulable

The question whether a 1-schedulable CFM is send-synchronizable over fully-matched traces is decidable.

Unmatched sends makes it harder



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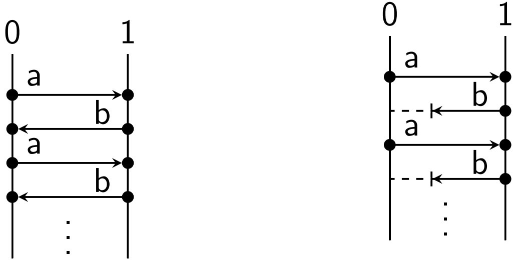

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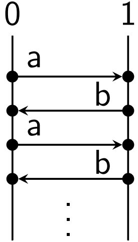

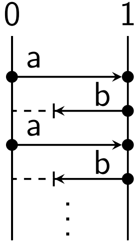

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How to account for unmatched sends?

Extended order

Order caused by "possibility of receive":

$a \dashrightarrow b$

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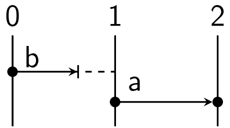

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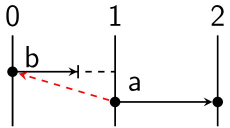

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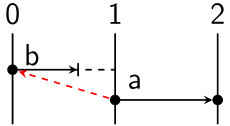

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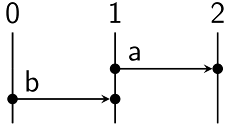
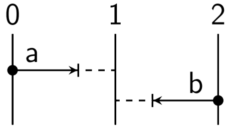
This order is convenient with SI!



Good traces

We say that $w \equiv_{\text{us}} w'$ if

- $w \equiv w'$
- The order of unmatched sends to each p is the same in w and w'



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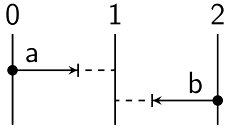

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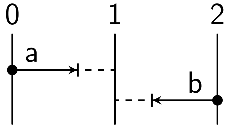

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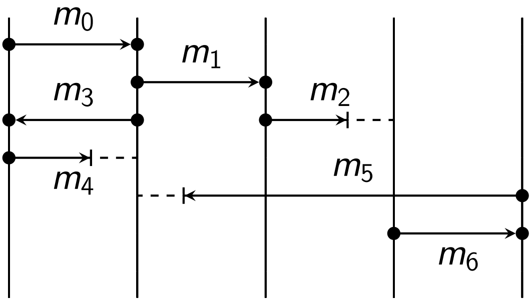

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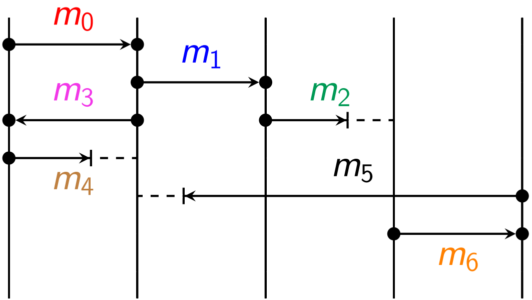

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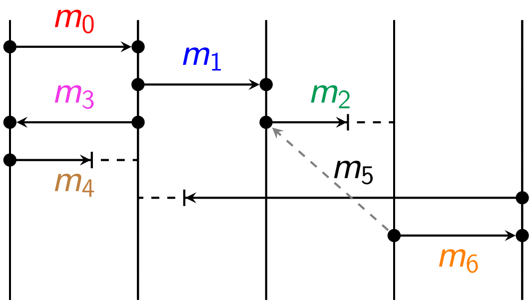

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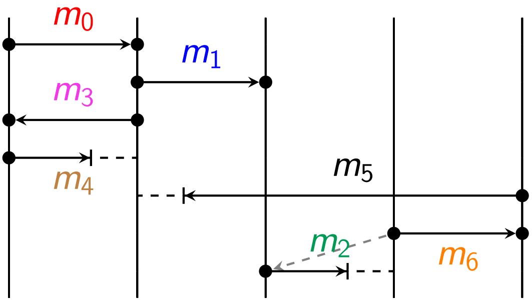

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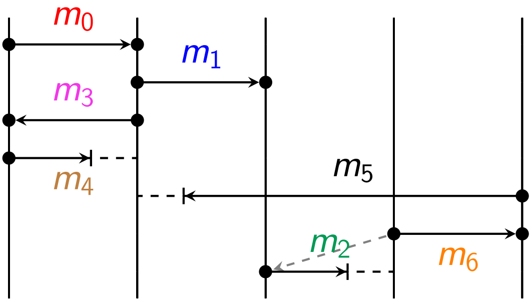

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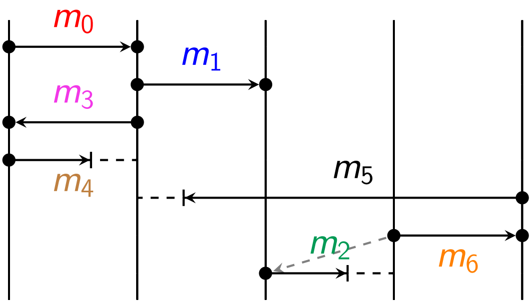

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Good traces & Send-synchronizability

If all traces of a CFM $\mathcal{A}$ are good:
$Tr(\mathcal{A})|_S = Cl_{SI}(\{w \mid w \text{ is a 1-scheduling of } \mathcal{A}\}|_S)$.

1 Introduction

2 Send-synchronizability

3 1-schedulability

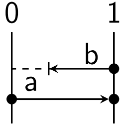
4 Fully-matched & Good traces

5 1-sched & Bad traces

6 Conclusion

Bad traces

Bad traces prevent send-synchronizability of 1-schedulable CFMs:



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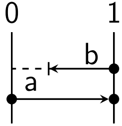

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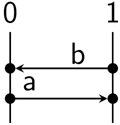

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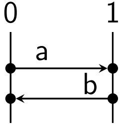

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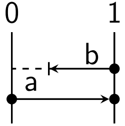

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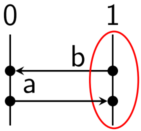

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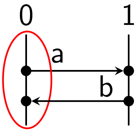
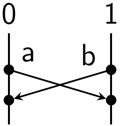


$\mathcal{A}$ will also have the following trace:



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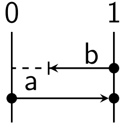

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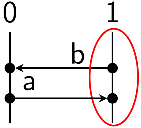

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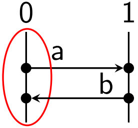
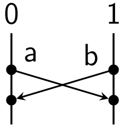


$\mathcal{A}$ will also have the following trace:
So $\mathcal{A}$ is not 1-schedulable!



Extended order: double unmatched

Recall:

$a \dashrightarrow b$

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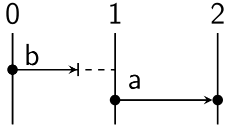

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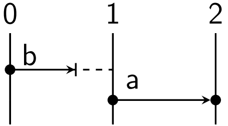

New order:

$a \ll^{w}_{\mathrm{us}} b$

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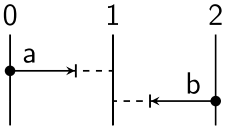

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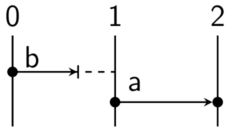

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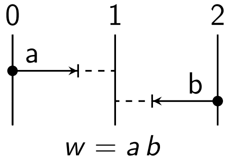

Rem: $u \equiv_{\mathrm{us}} v$ iff $\ll^{u}_{\mathrm{us}} = \ll^{v}_{\mathrm{us}}$.

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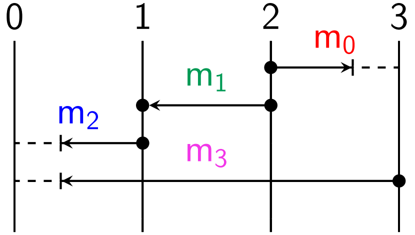

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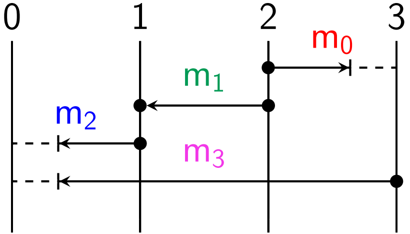

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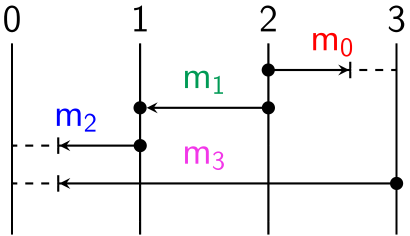

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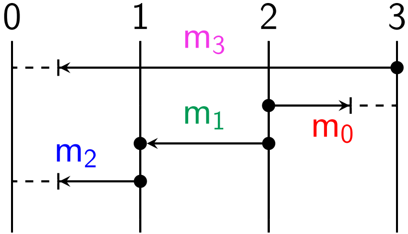

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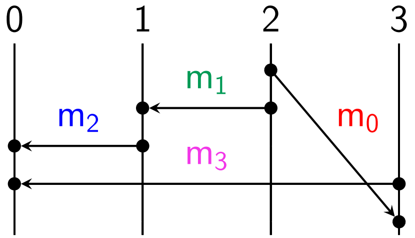

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$s_0 <_{\mathbb{P}} s_1 \ \mathrm{msg}\ r_1 <_{\mathbb{P}} s_2 \ll^{w}_{\mathrm{us}} s_3 \dashrightarrow s_0$

$w' = s_3\ s_0\ s_1\ r_1\ s_2$.

Detecting bad traces

A trace w is good if some 1-scheduling $w \equiv_{\mathrm{us}} w'$ exists with no $\dashrightarrow$-backward arcs

A trace w is **bad** iff it has a $(<_{\mathbb{P}} \cup <_{\mathtt{mb}} \cup \mathrm{msg} \cup \dashrightarrow \cup \ll^{w}_{\mathrm{us}})$-cycle

Bad traces and send-synchronizability

If a 1-schedulable CFM has some bad trace, then it is not send-synchronizable.

Checking if a 1-schedulable CFM has some bad trace is PSPACE-complete.

$s_0 <_{\mathbb{P}} s_1 \;\mathrm{msg}\; r_1 <_{\mathbb{P}} s_2 \ll^{w}_{\mathrm{us}} s_3 \dashrightarrow s_0$

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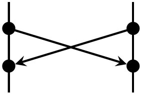

A 2-exchange.

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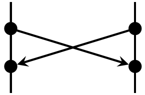

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THANK YOU

CFM for Pre-MPCP reduction