High-Level Message Sequence Charts

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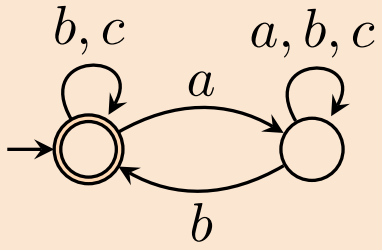
based on joint work with **Benedikt Bollig** and **Paul Gastin**

PaVeDyS meeting, May 13, 2025

Regular Languages of Words

[Kleene 1956]

Finite automata



Regular Expressions

$$((a + b + c)^* b + c)^*$$

[Büchi 1960,
Elgot 1961
Trakhtenbrot 1962]

$$L = \{aab, bcc, acabc, \cdots\}$$

Monadic Second-Order Logic

$$\forall x.\, a(x) \implies \exists y.\, x < y \wedge b(y)$$

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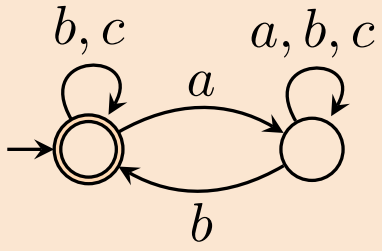

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What about **concurrent** systems?

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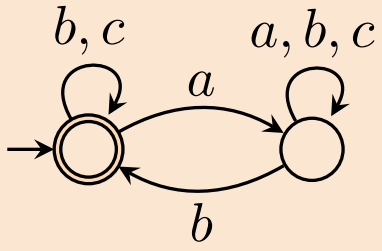

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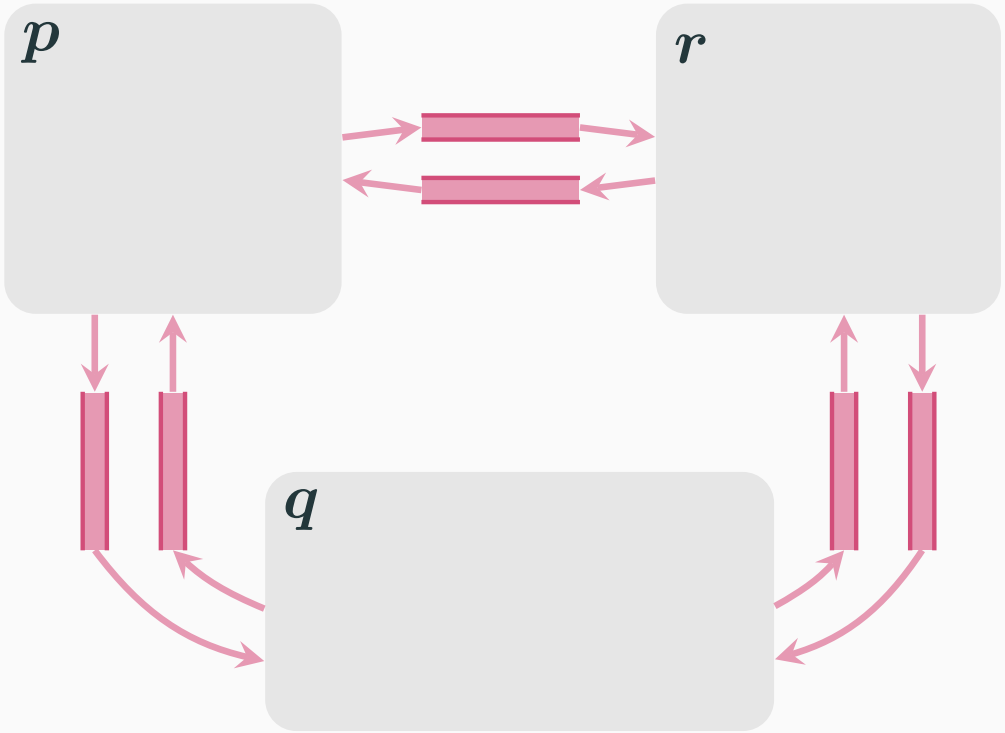
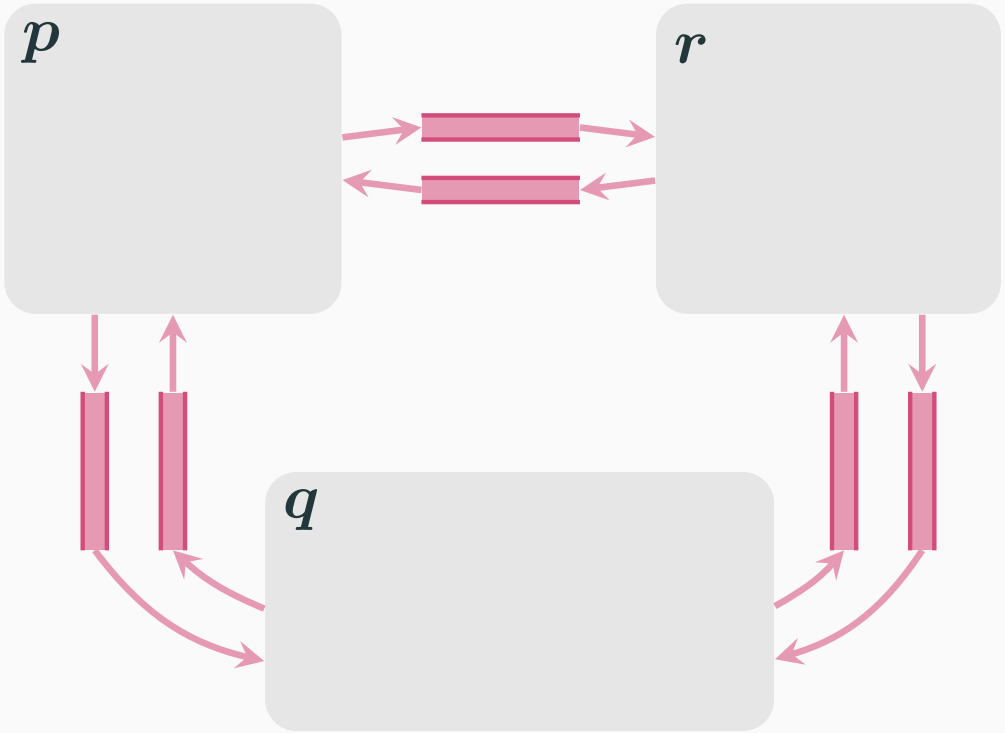
What about **communicating automata**?

Communicating finite-state machines (CFMs)[1]



[1] [Brand, Zafiropulo 1983]

Communicating finite-state machines (CFMs)[1]



Fixed, finite set of processes, e.g. $\{p, q, r\}$

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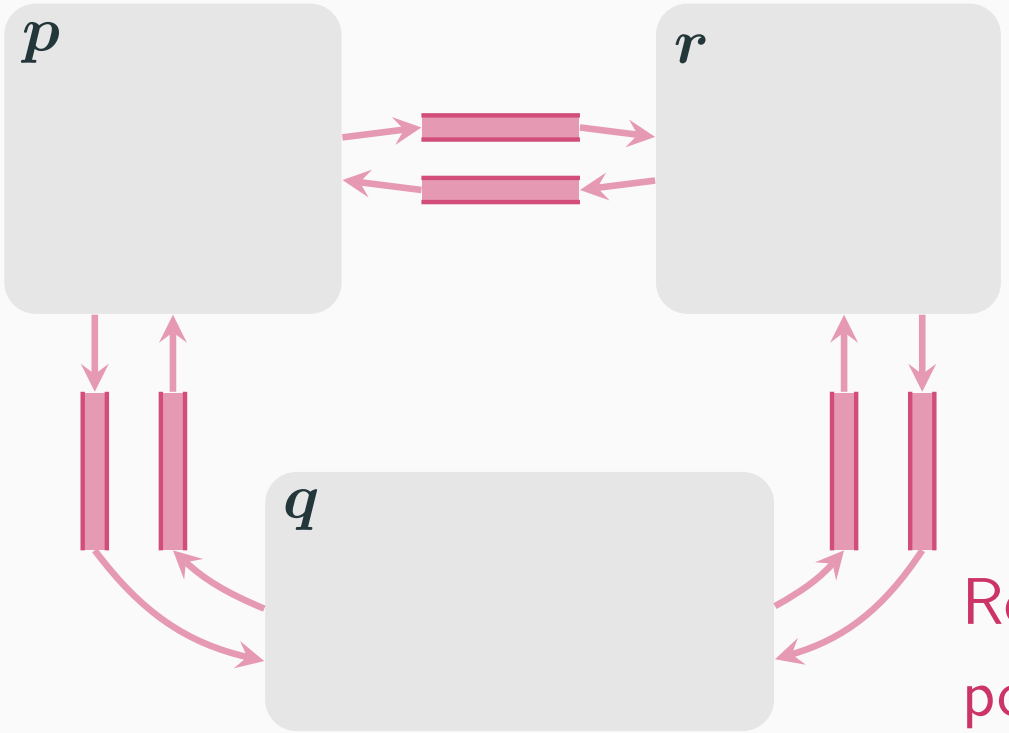

Reliable unbounded
point-to-point FIFO channels

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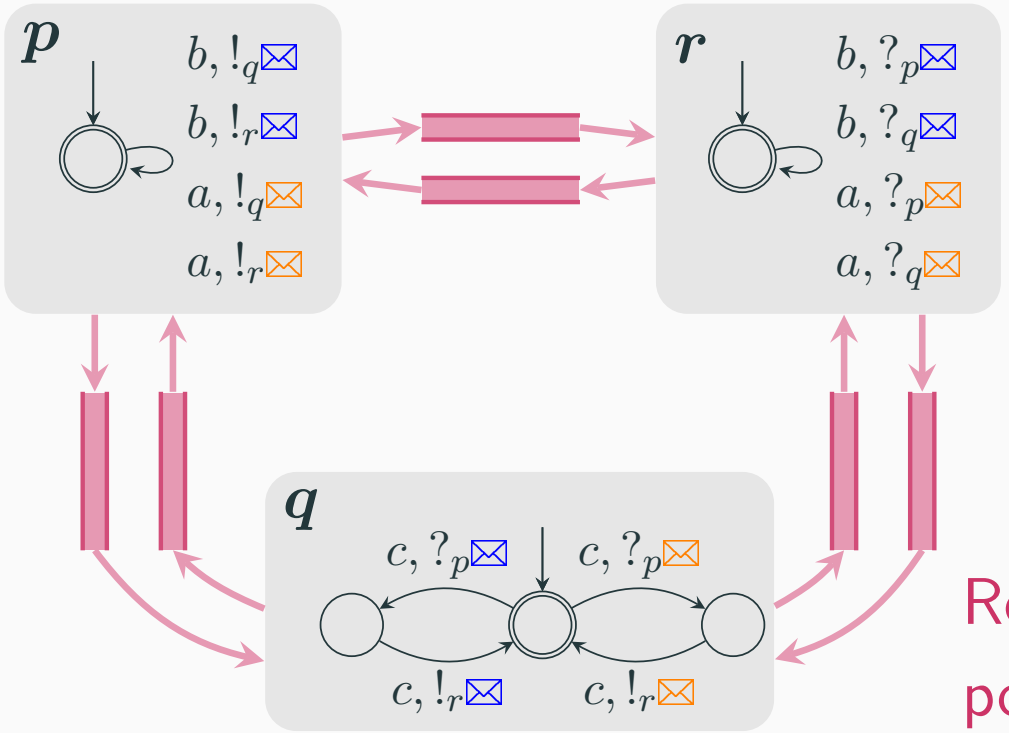

One finite automaton for each process

- Finite input alphabet, e.g. $\Sigma = \{a, b, c\}$
- Sends/receives from a finite message alphabet, e.g. $\{$✉, ✉$\}$

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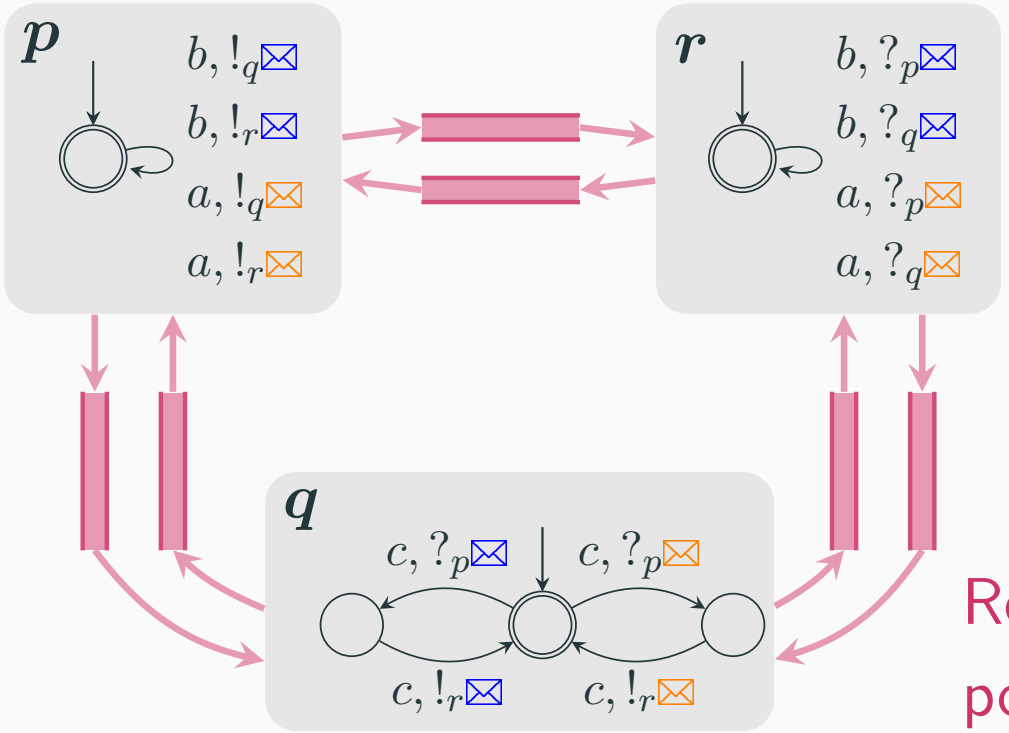

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Global acceptance condition

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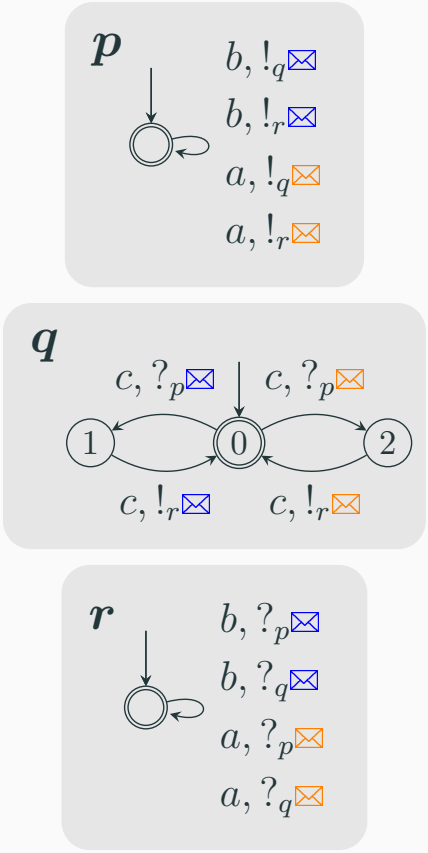
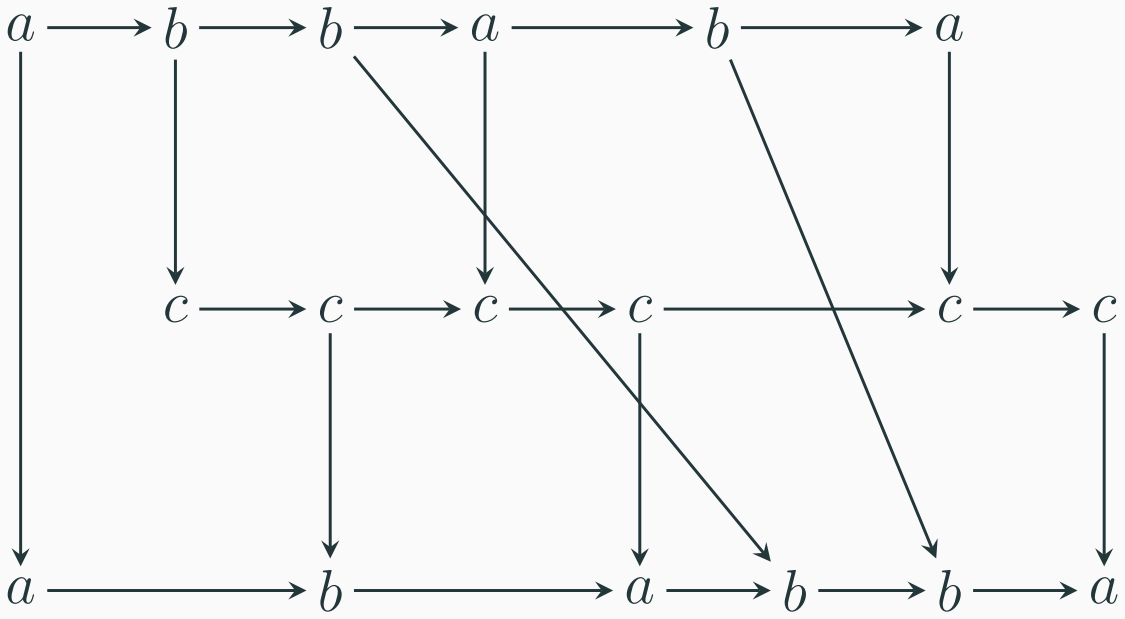
Message Sequence Charts (MSC)

The language of a CFM is a set of Message Sequence Charts.





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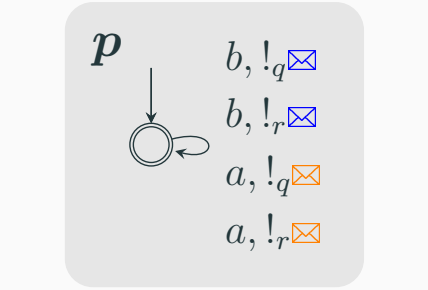

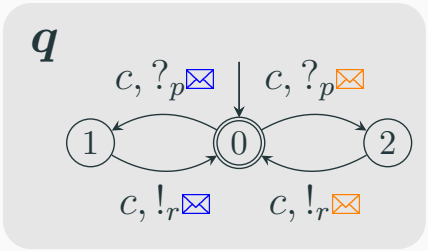

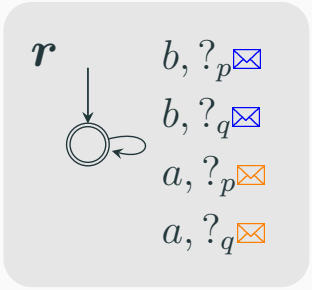

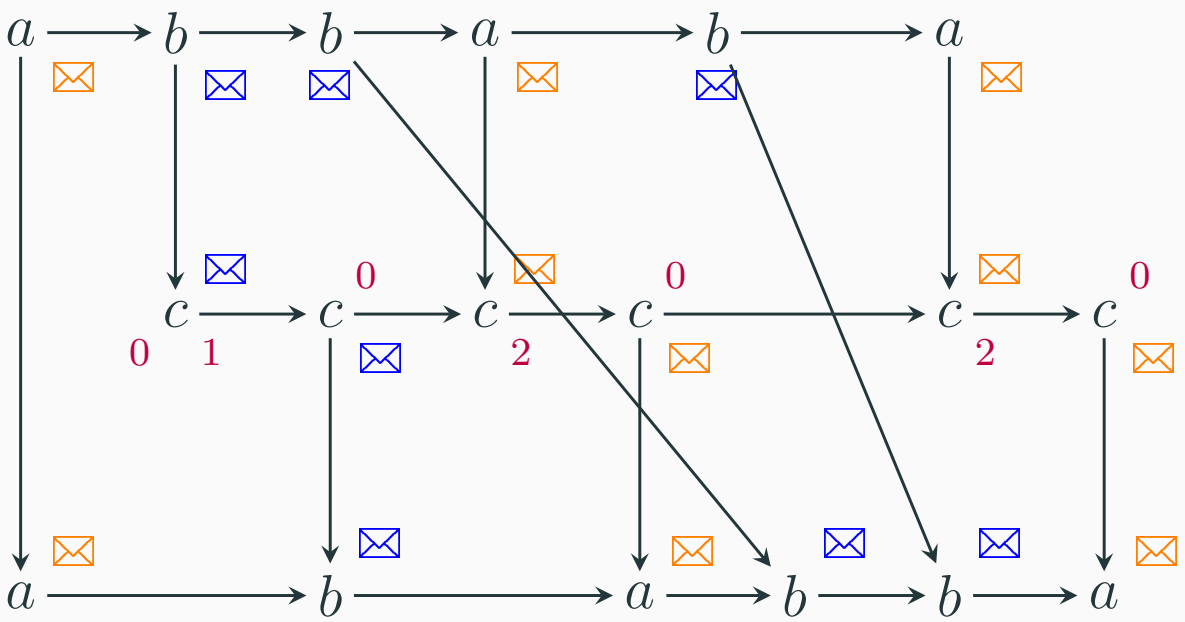

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CFM with ≥ 2 processes can easily simulate Turing machines

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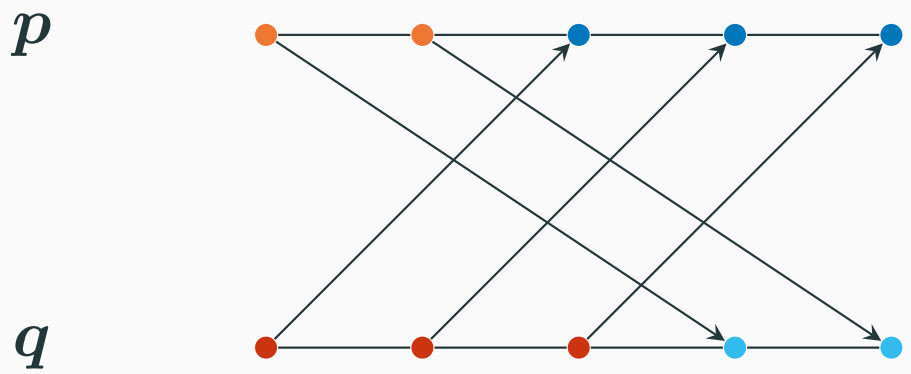
In this talk: mainly unbounded, sometimes bounded

Linearizations and bounded MSCs



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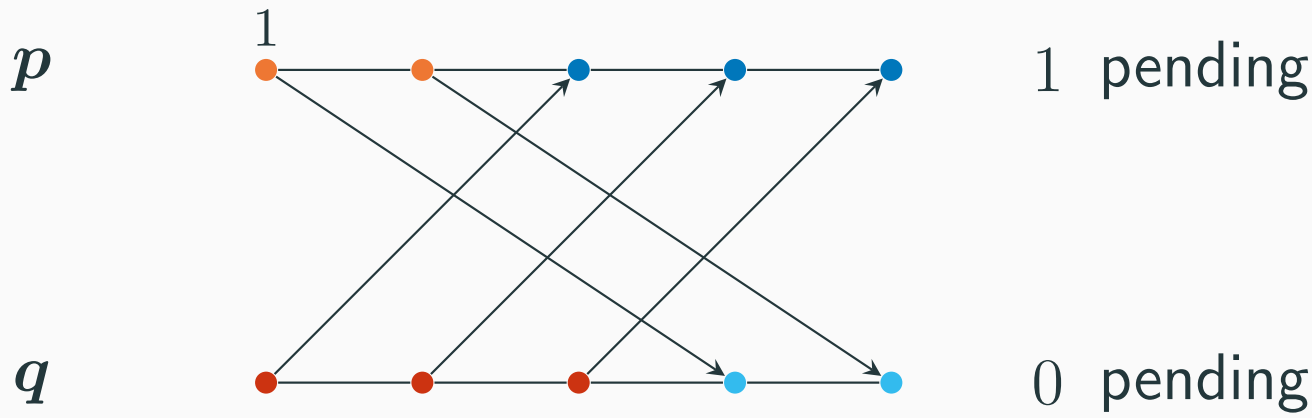

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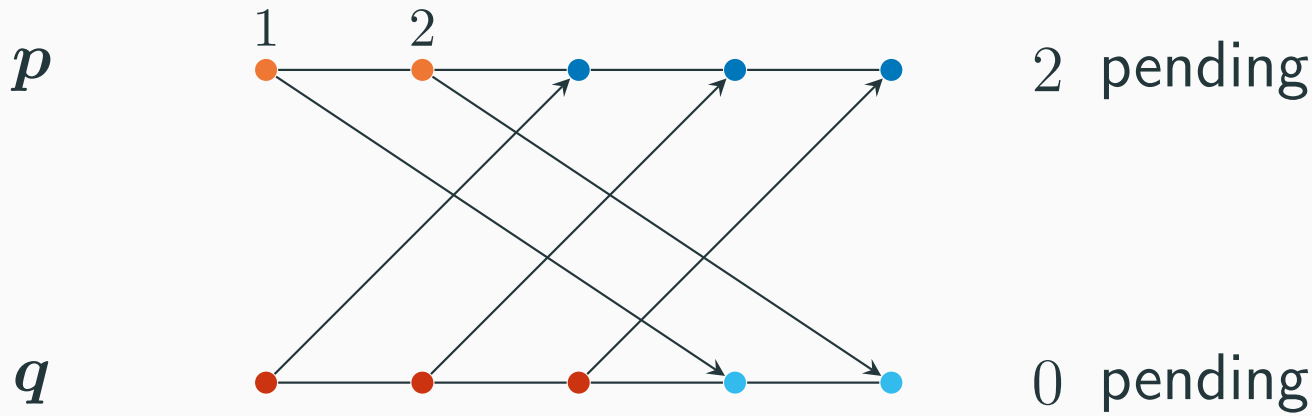

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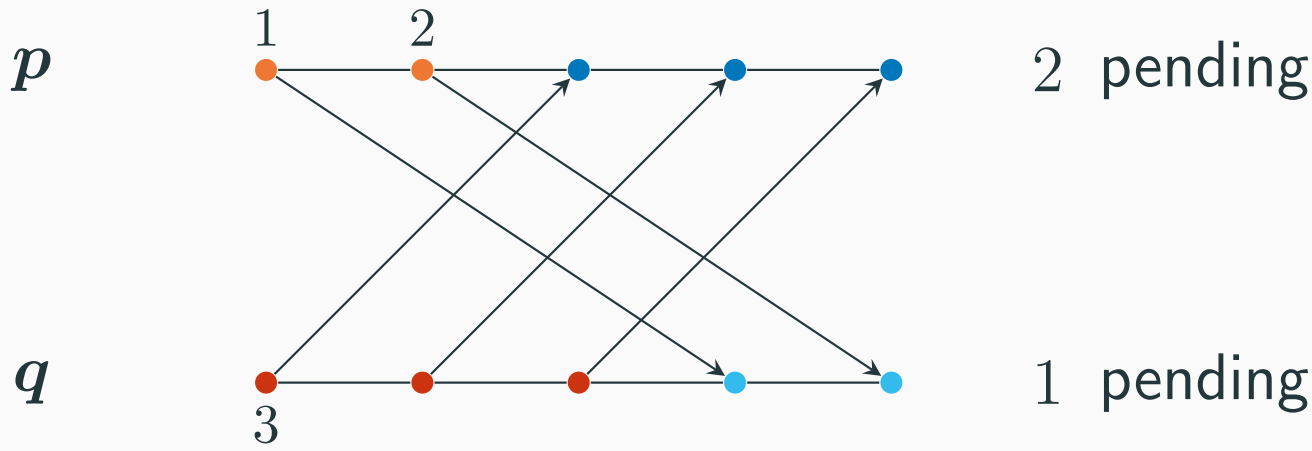

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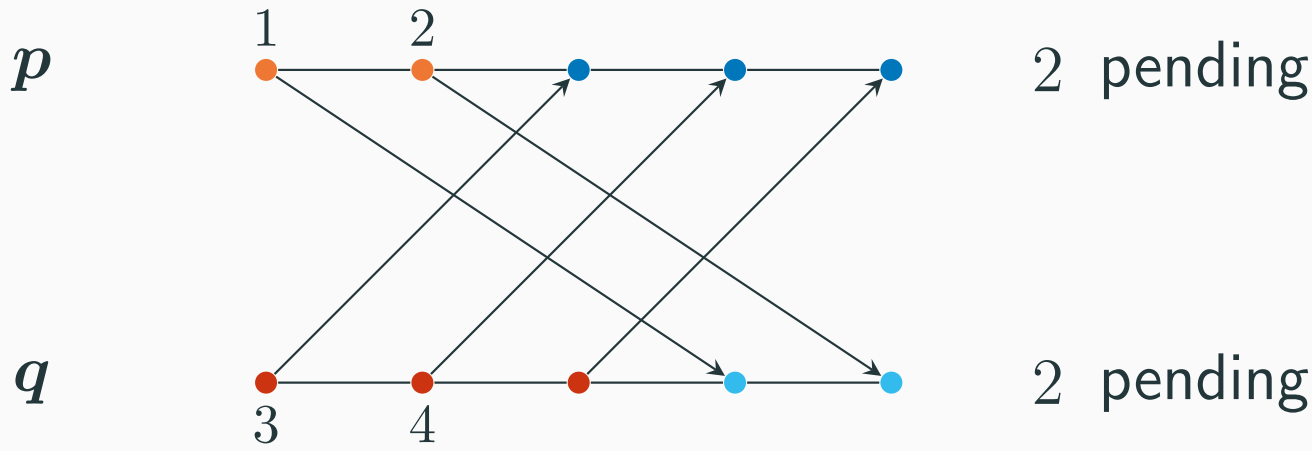

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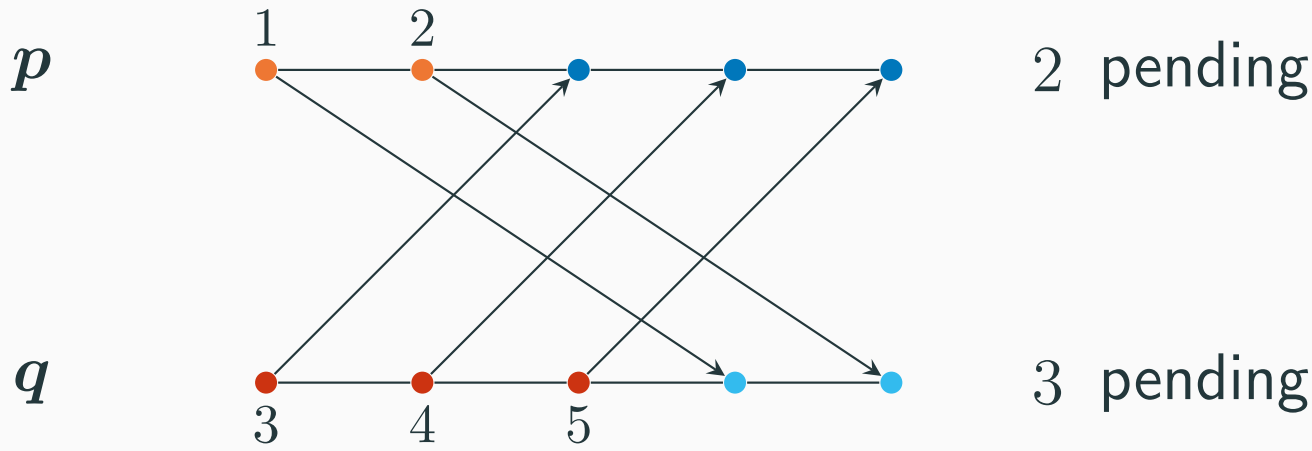

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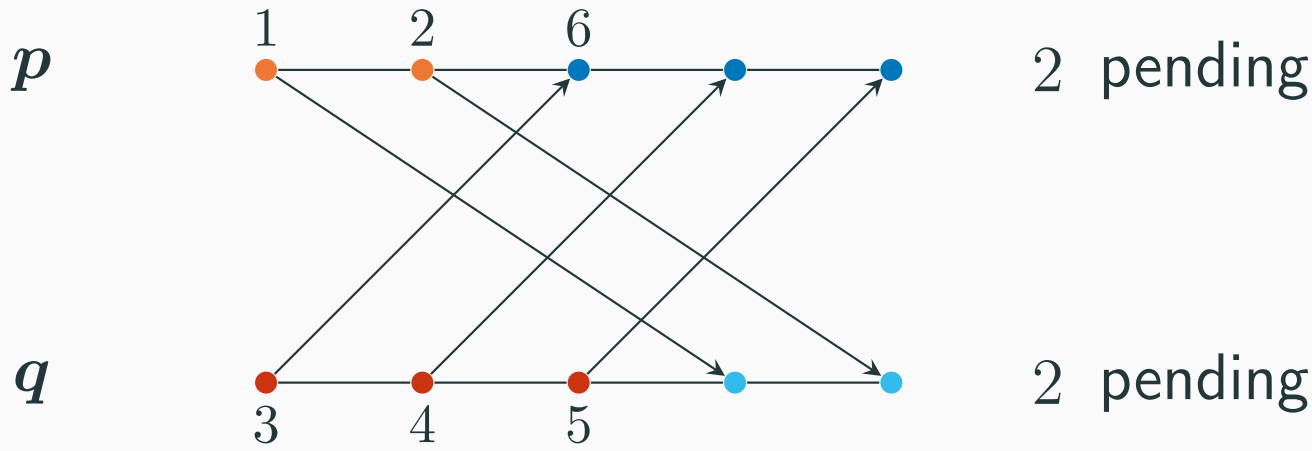

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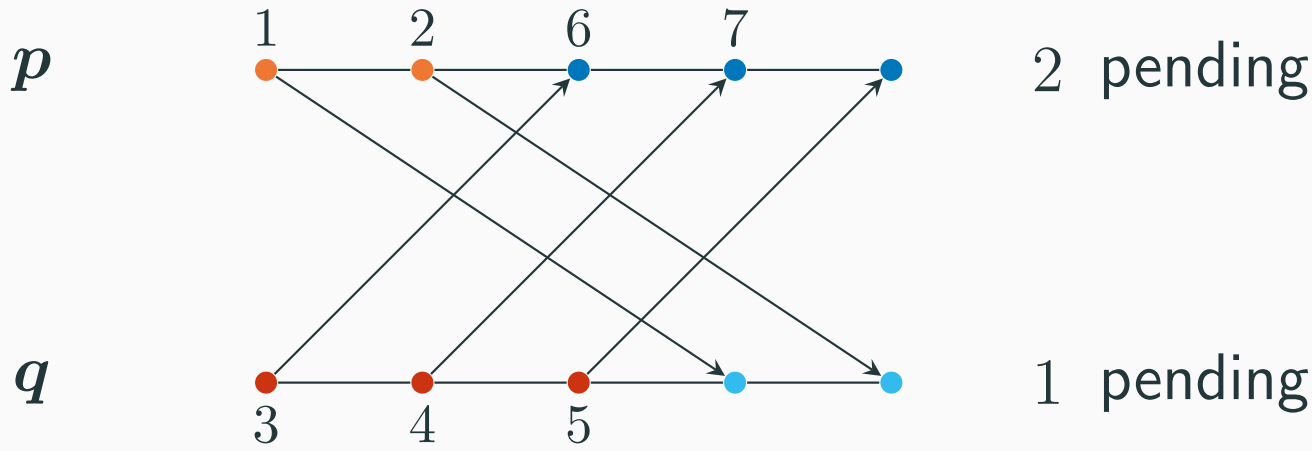

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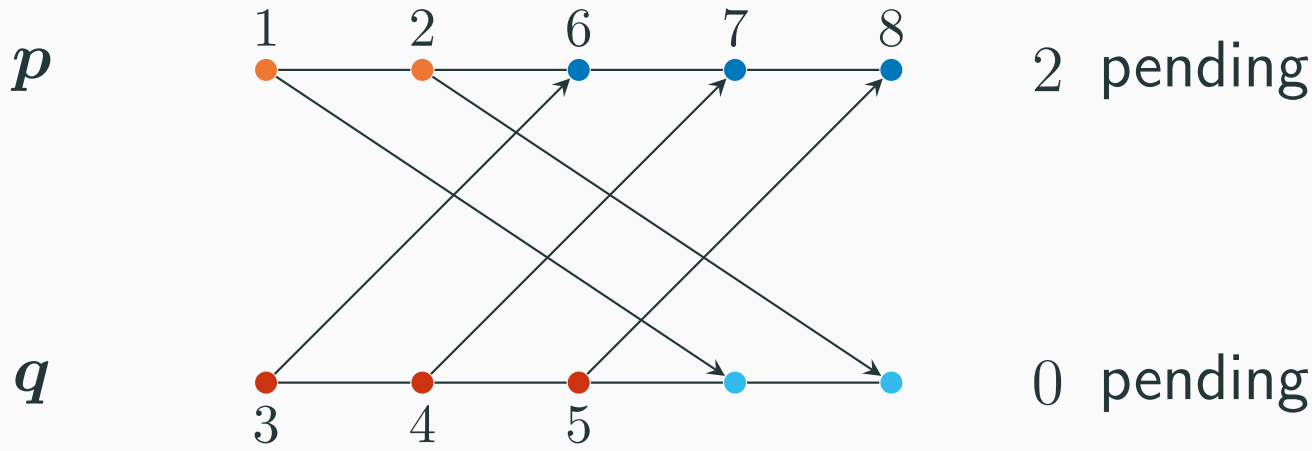

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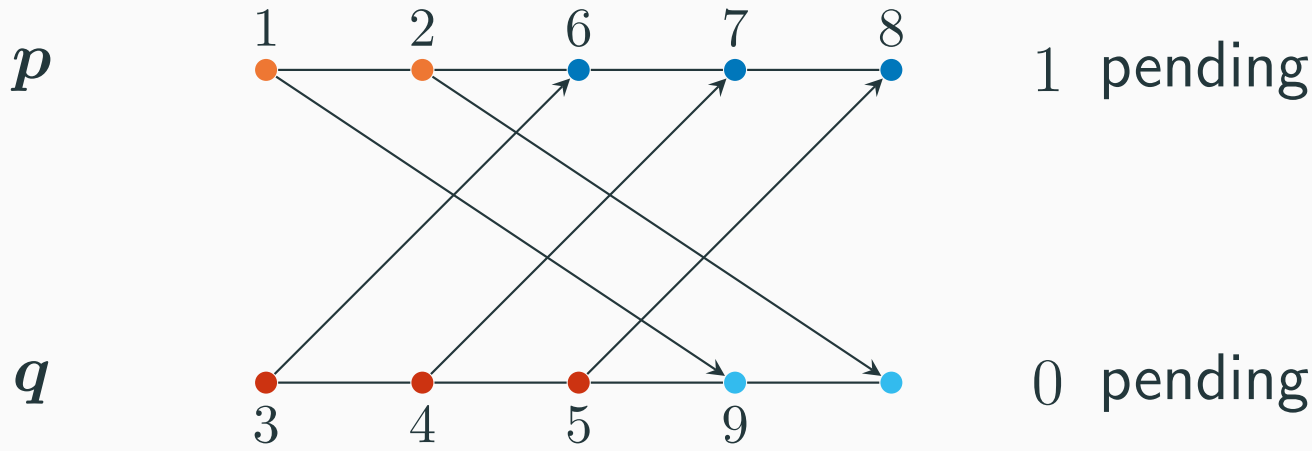

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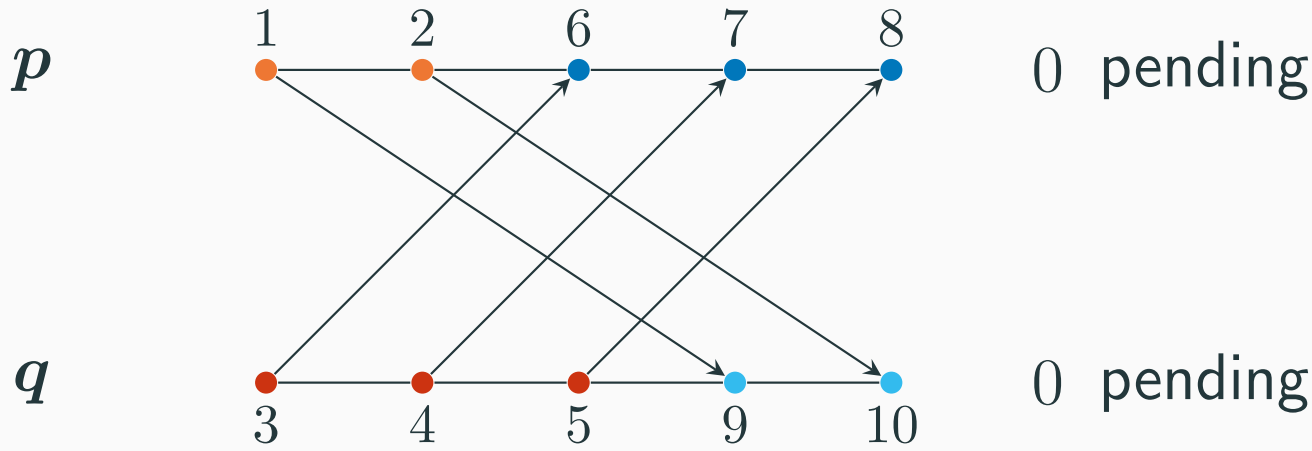

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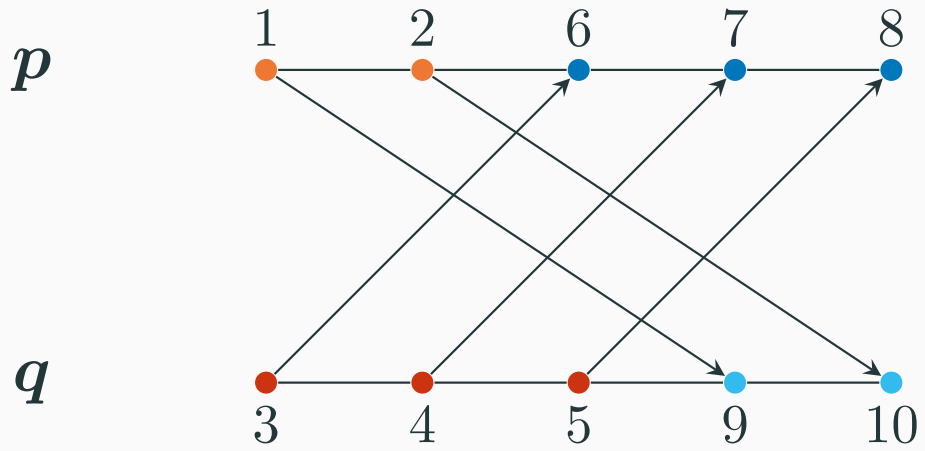

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B-bounded = at most B pending messages in each channel

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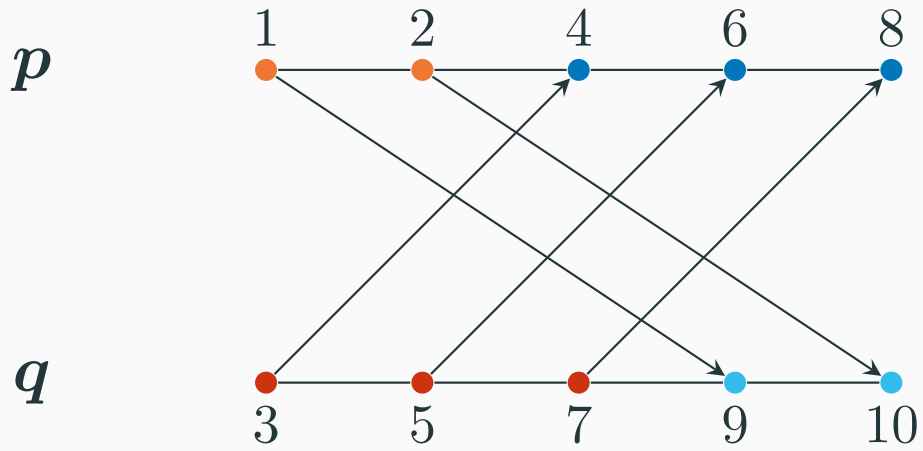

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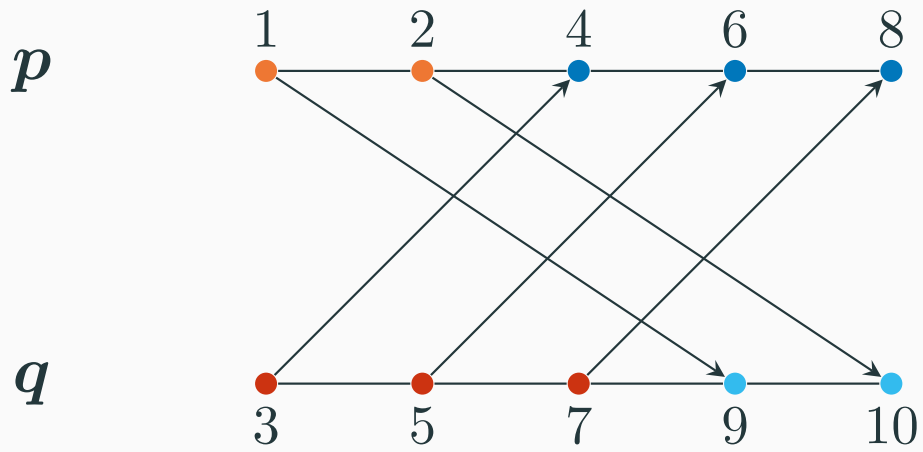

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$\ldots$

M is $\exists B$-bounded if at least one linearization is B-bounded

$\forall B$-bounded if all linearizations are B-bounded

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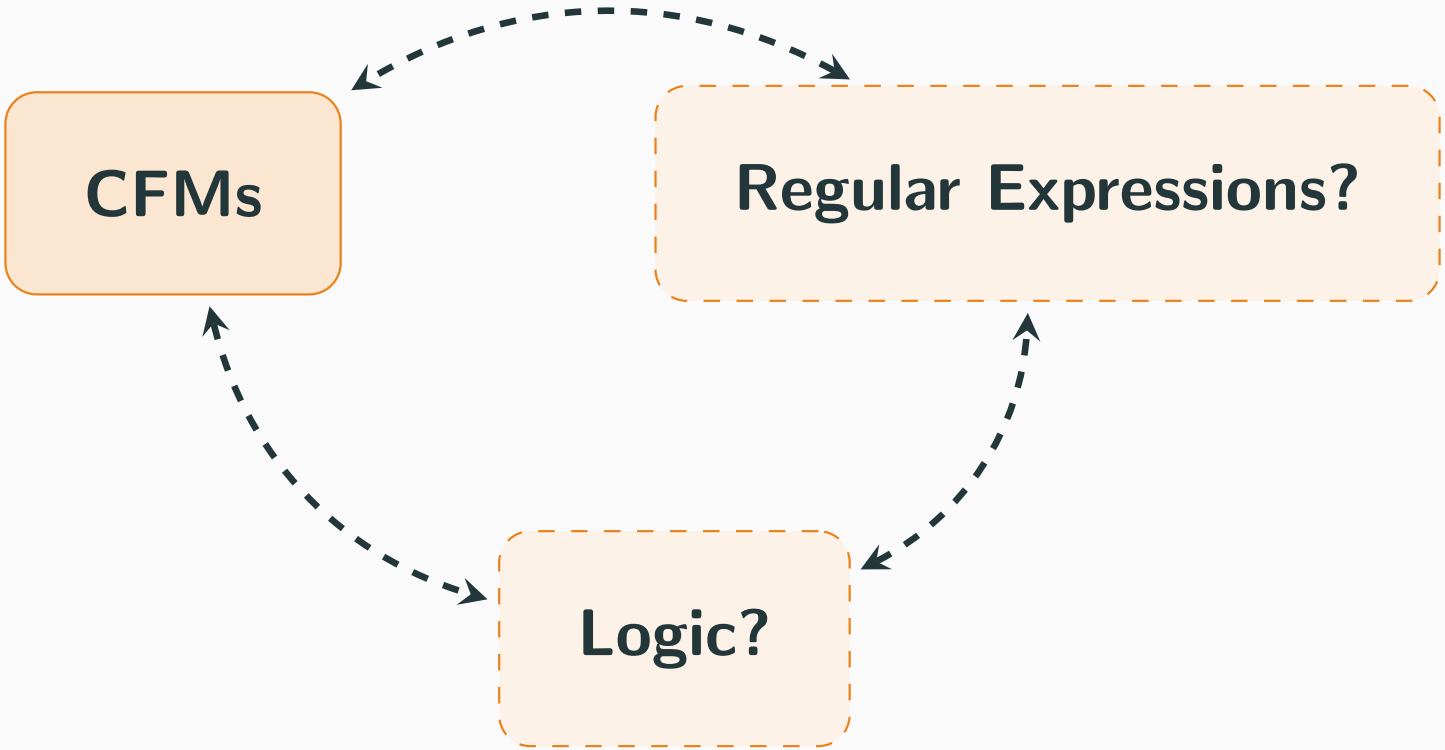

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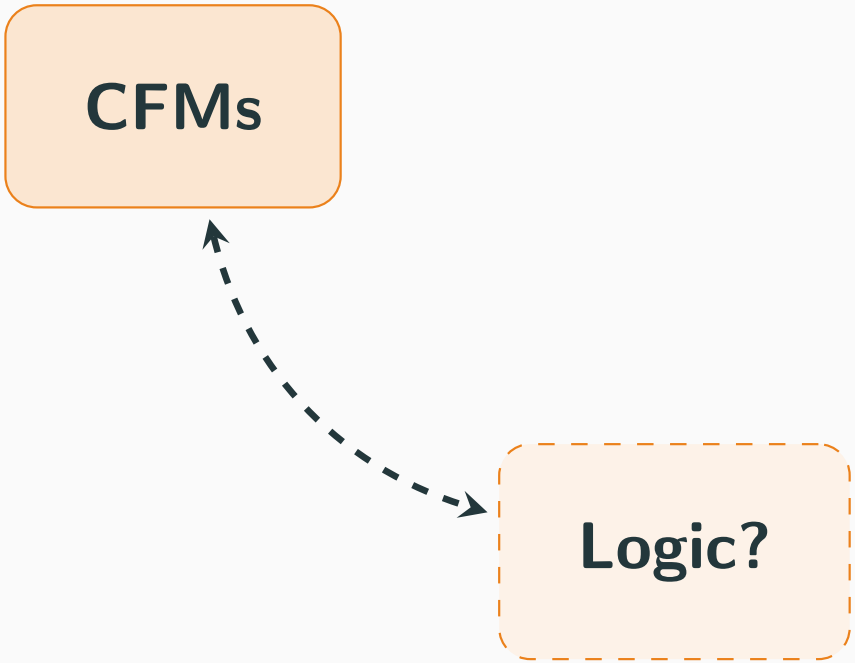
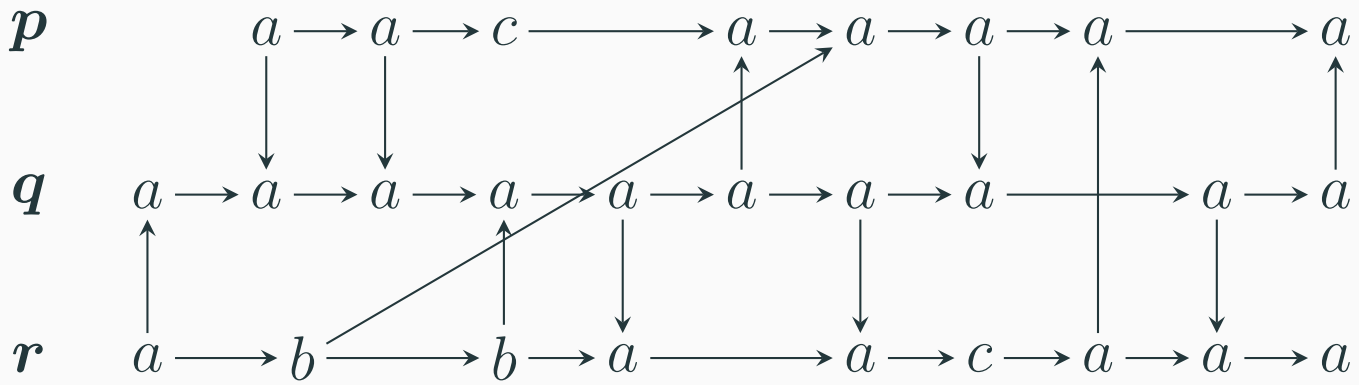


Monadic Second-Order logic (MSO) over MSCs

$\varphi ::=$



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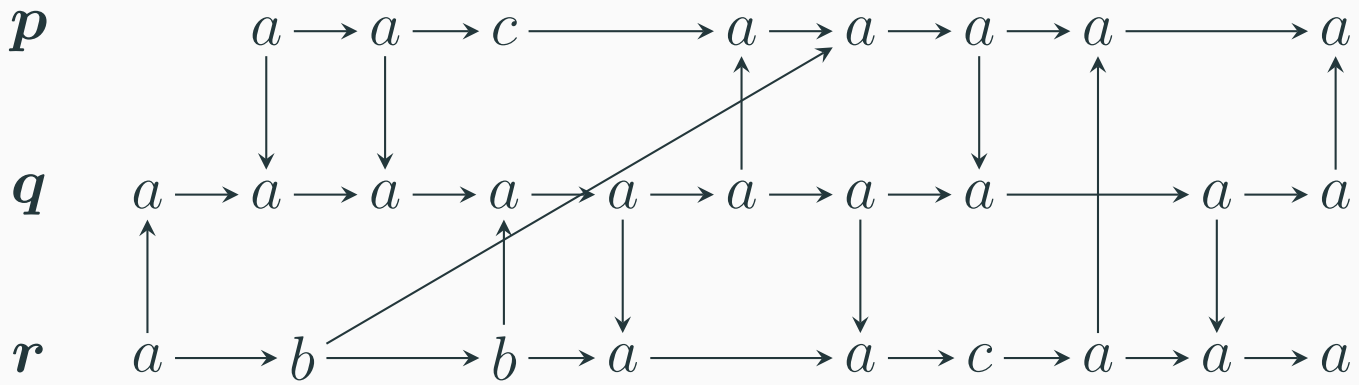
$\varphi ::= \quad a(x) \mid p(x)$ label/process of event x



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$$\varphi ::= \quad a(x) \mid p(x) \qquad \text{label/process of event } x$$
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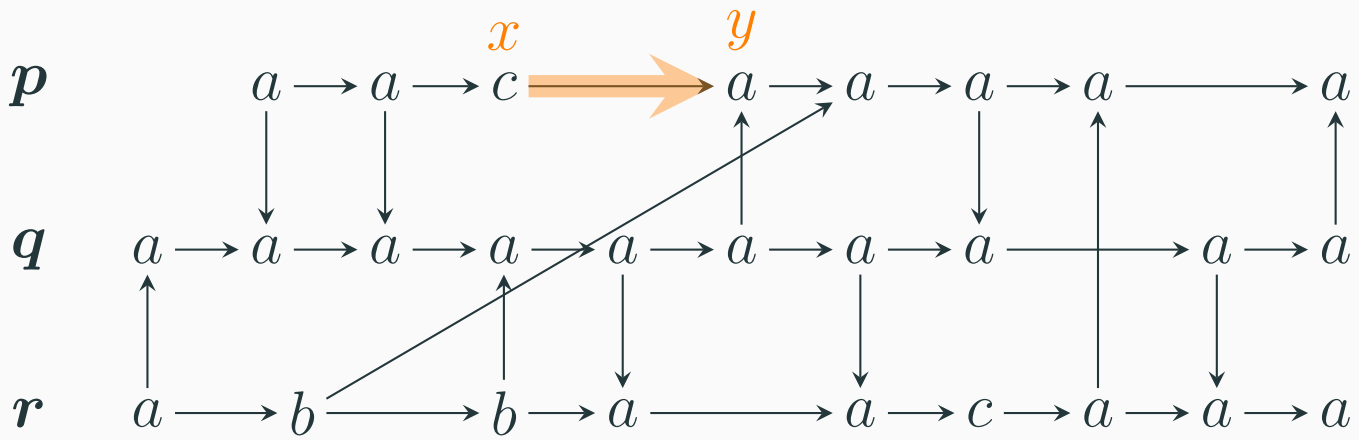

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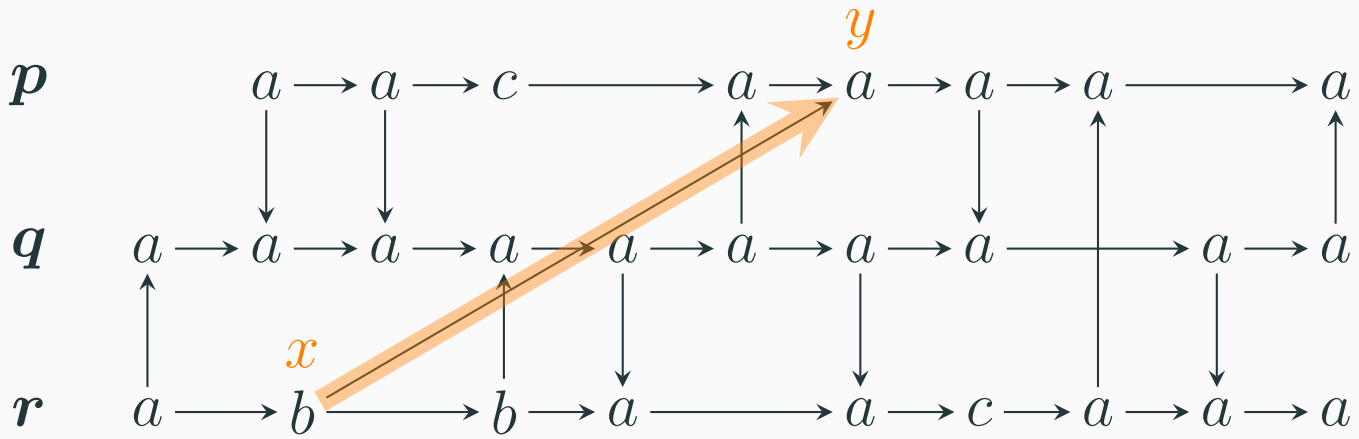

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$\varphi ::=$	$a(x) \mid p(x)$	label/process of event x
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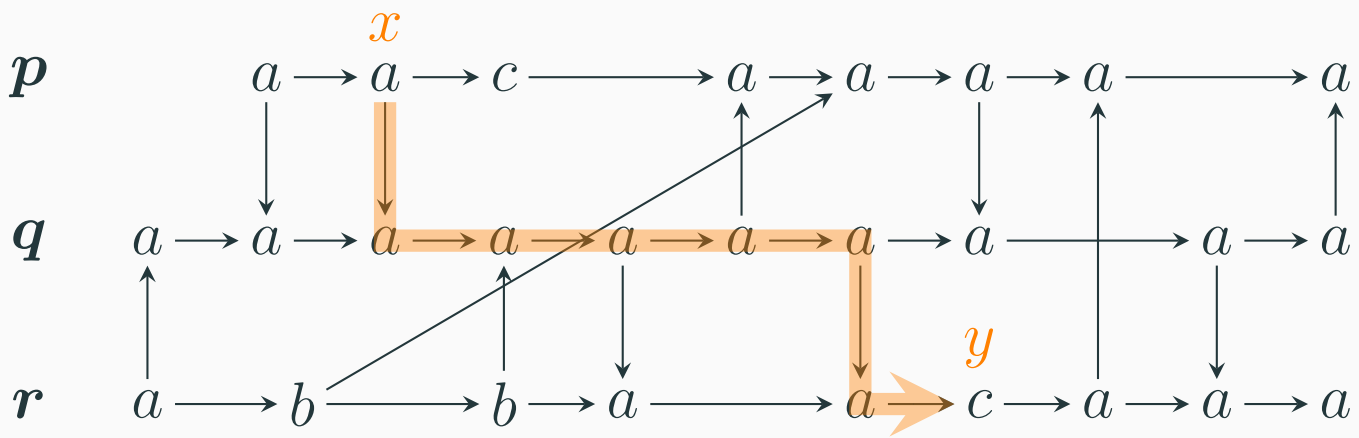

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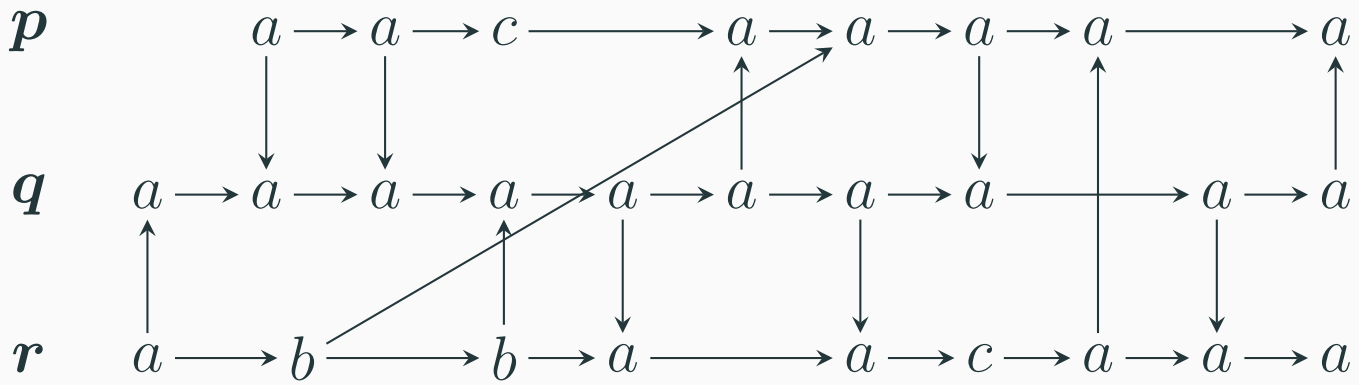

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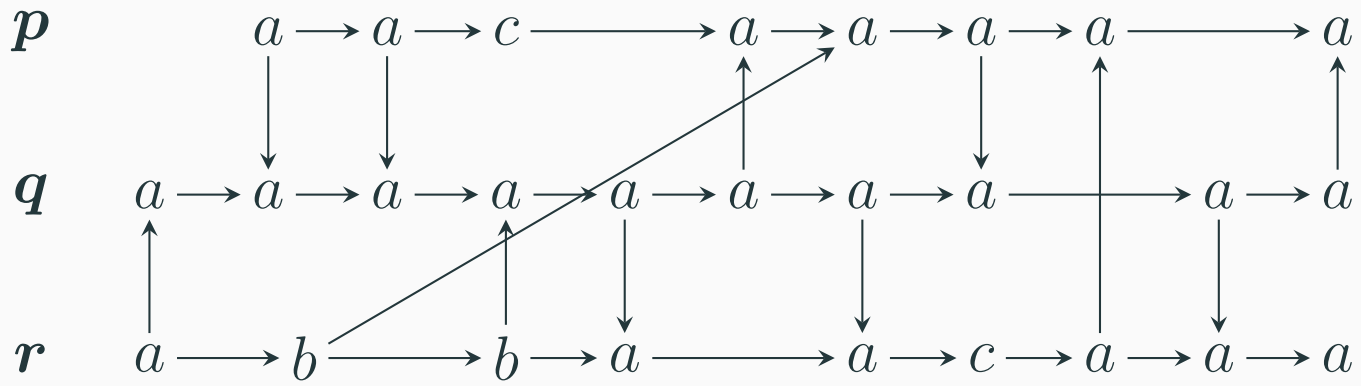

Example: mutual exclusion $\neg(\exists x.\ \exists y.\ c(x) \wedge c(y) \wedge\ x \parallel y\ )$

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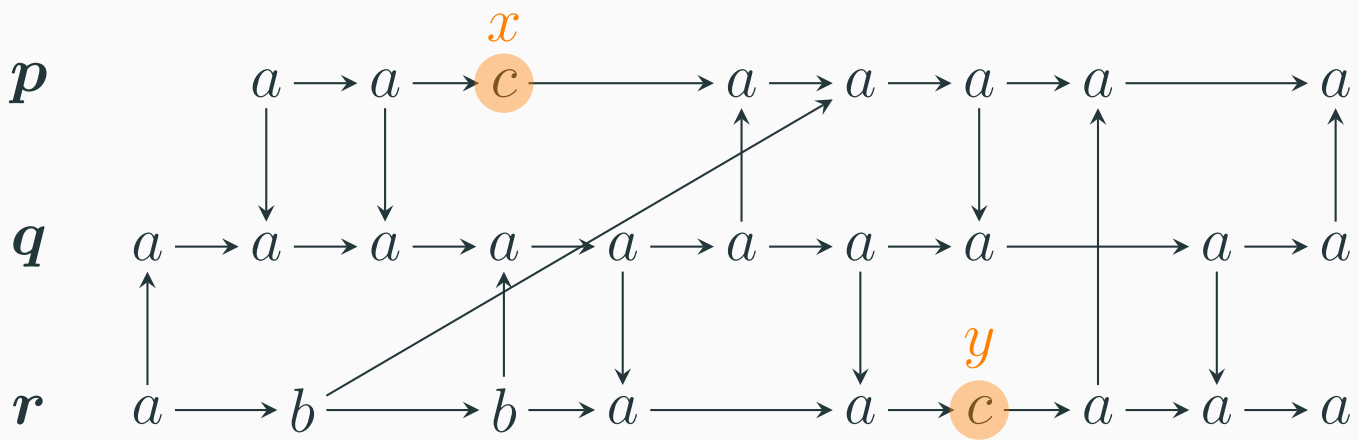

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$\neg(x \leq y) \wedge \neg(y \leq x)$ ⟵

Let's get back to words...

MSO $\rightarrow$ Automata

Inductive translation:

- $\ldots$
- Conjunction $\rightarrow$ Product
- Disjunction $\rightarrow$ Union
- Existential quantification $\rightarrow$ Projection
- Negation $\rightarrow$ Complement

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Theorem (Bollig, Leucker 2006)
CFMs are **not** closed under complement.
Thus CFM $\neq$ MSO$[\leq, \lhd]$.

Logical characterization of CFMs

Theorem (Bollig, Leucker 2006)
Over finite MSCs, CFM $=$ EMSO$[\lhd, \rightarrow] \subsetneq$ MSO$[\lhd, \rightarrow]$.

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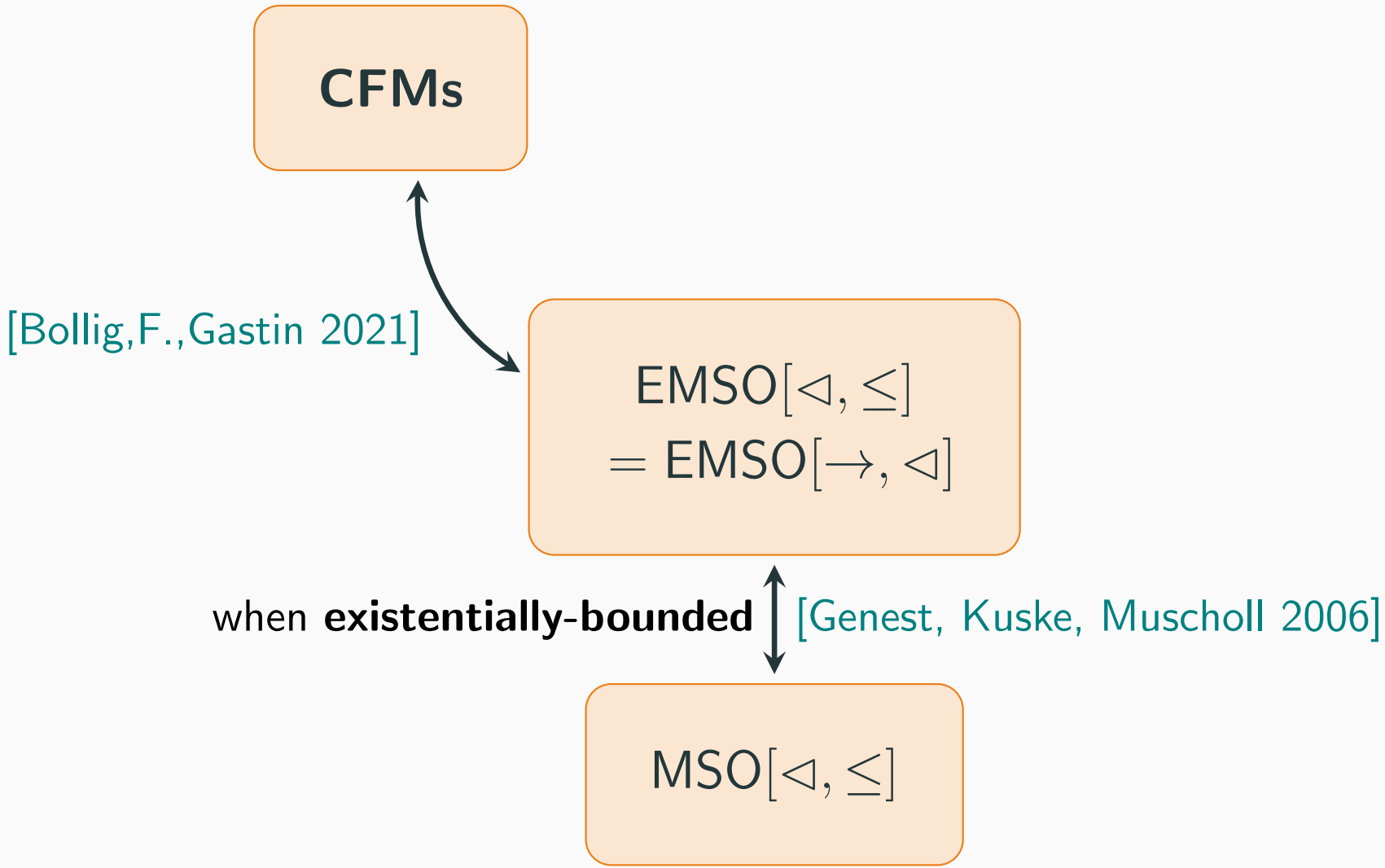
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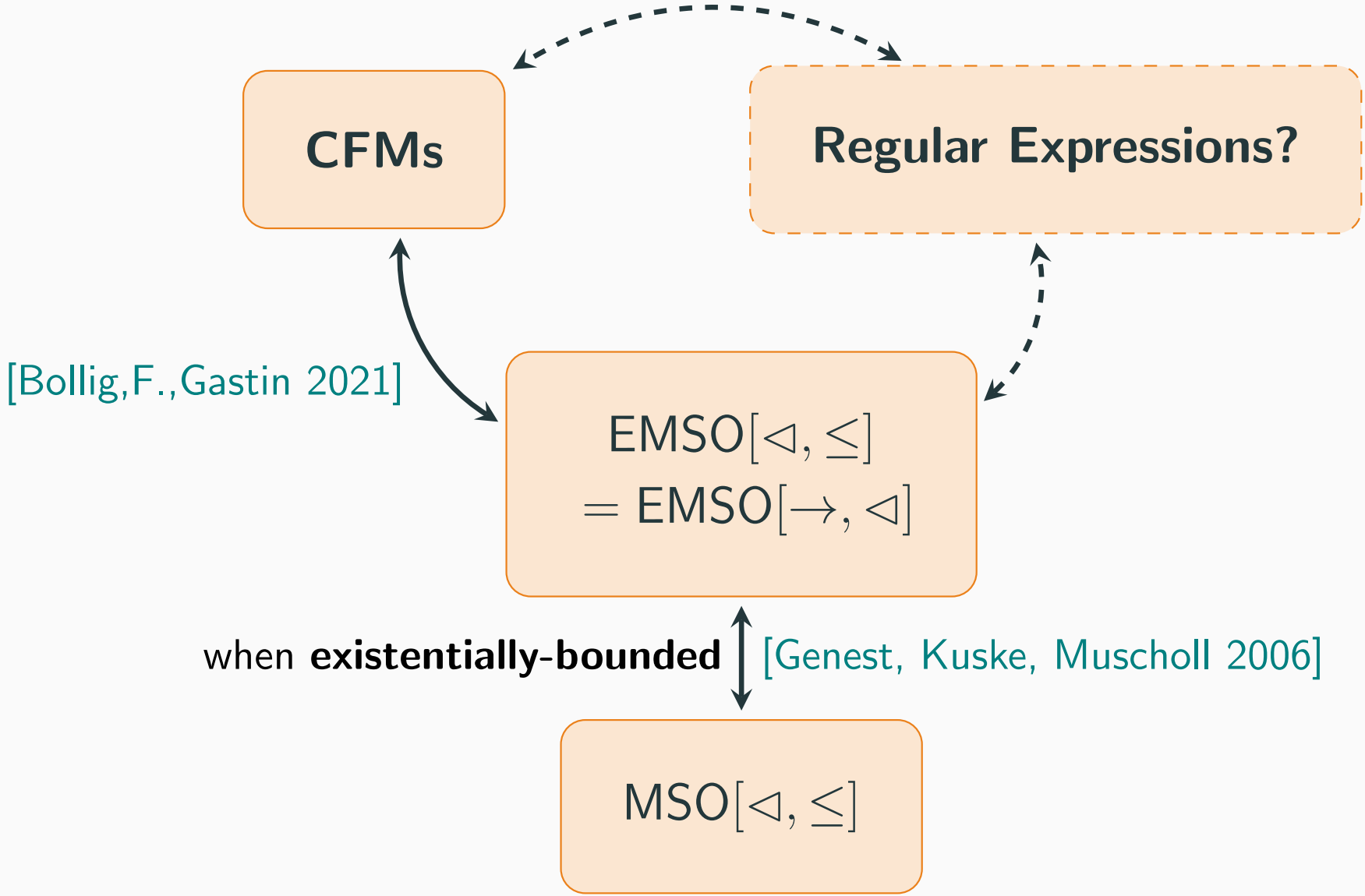
CFMs
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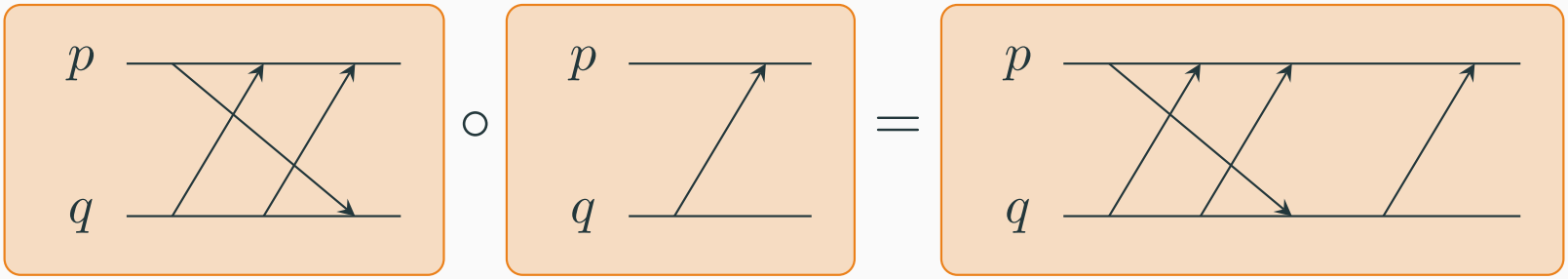
Concatenation and compositional MSCs



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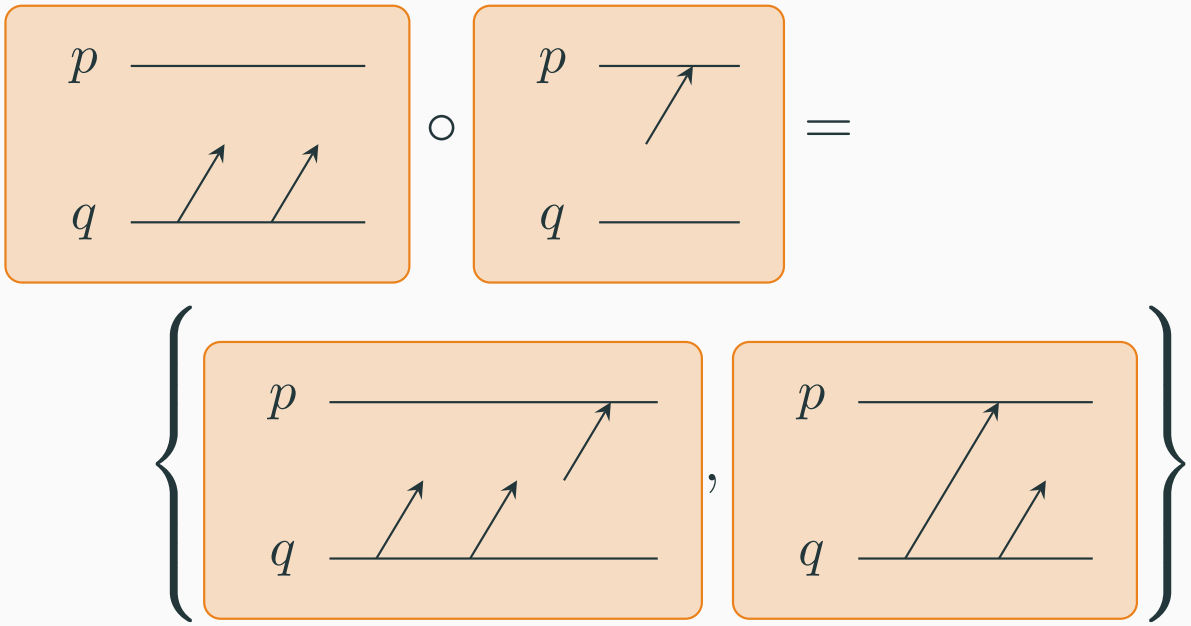

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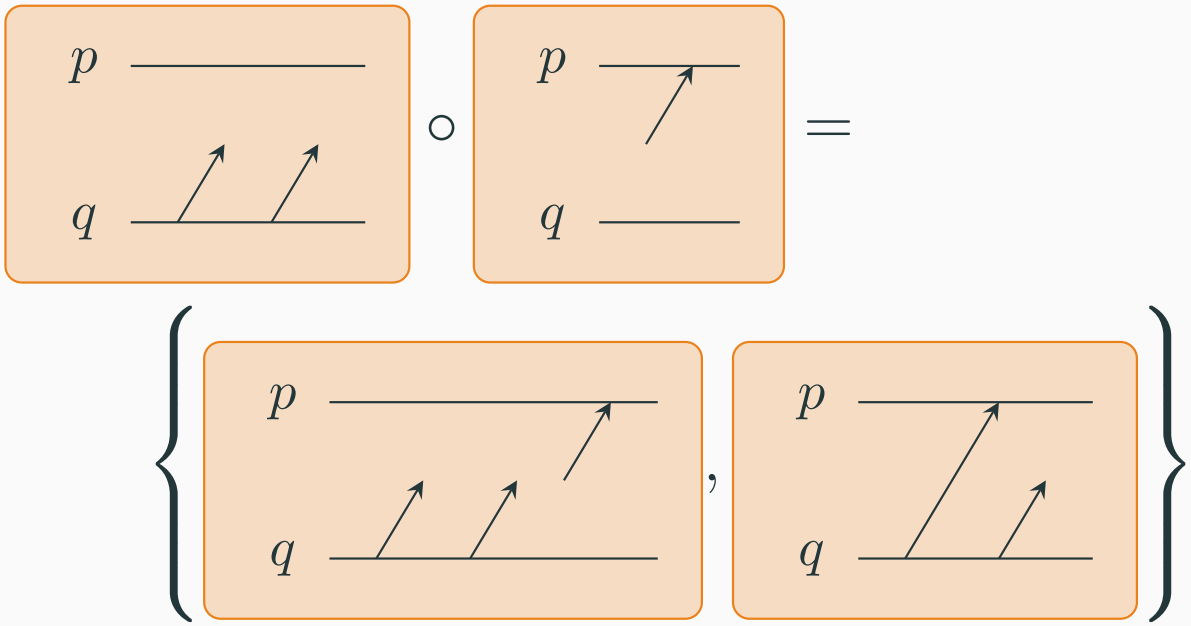
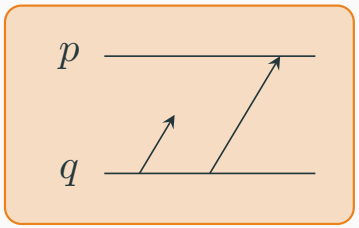
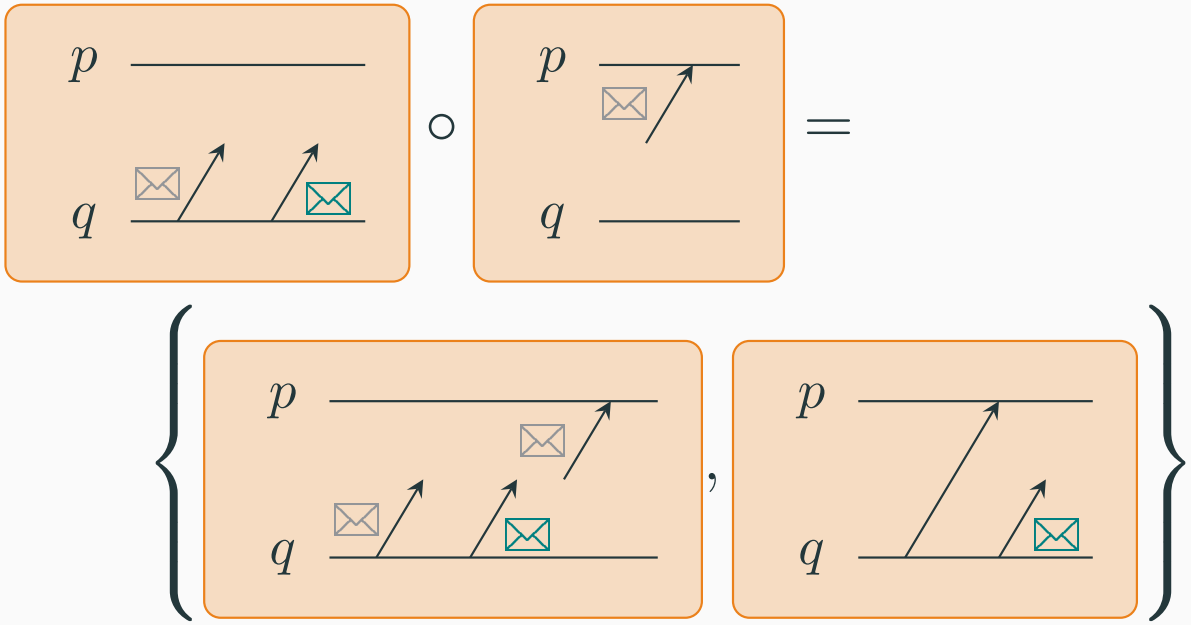


NB:  is not a compositional MSC (not FIFO)

Concatenation and compositional MSCs



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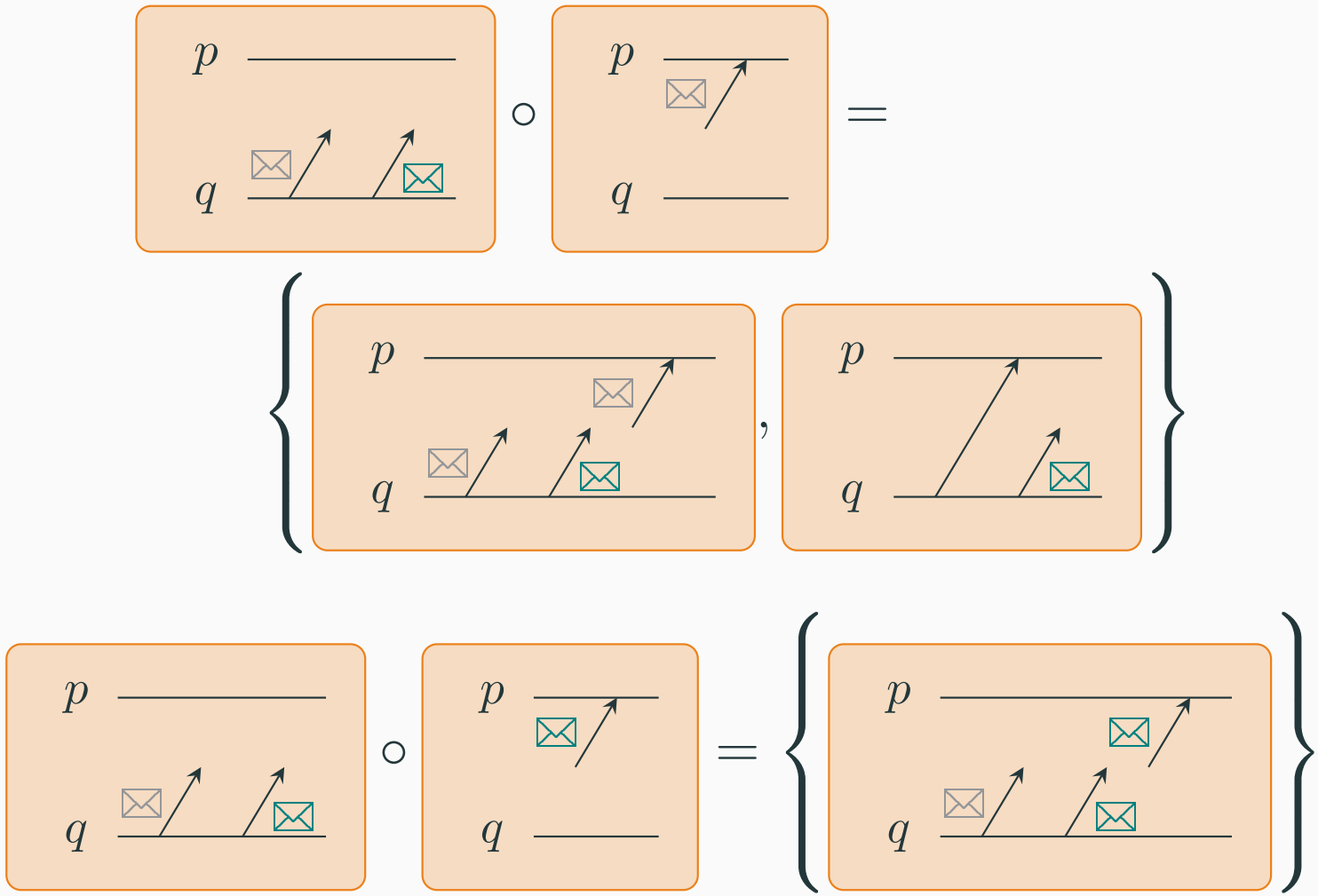
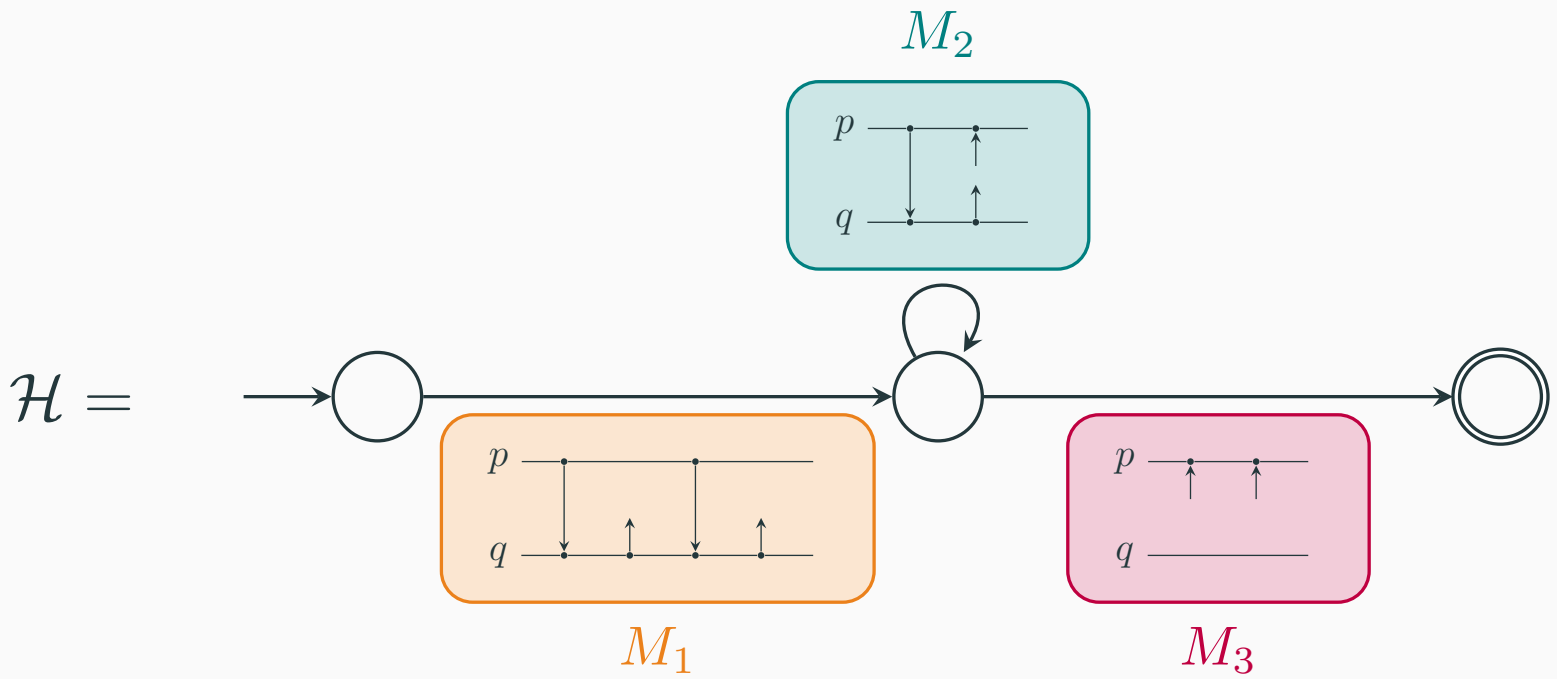


High-level message sequence charts (HMSCs)

(or message sequence graphs (MSGs))



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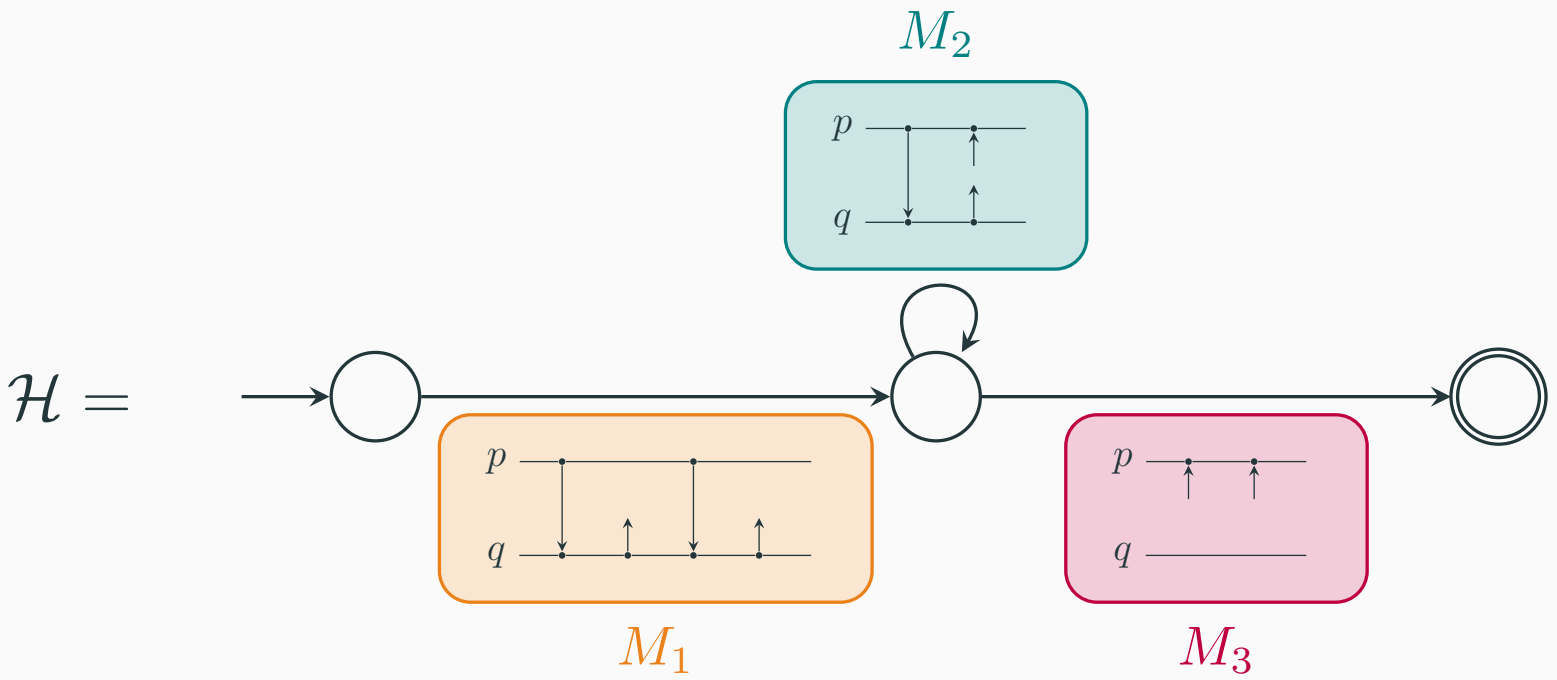
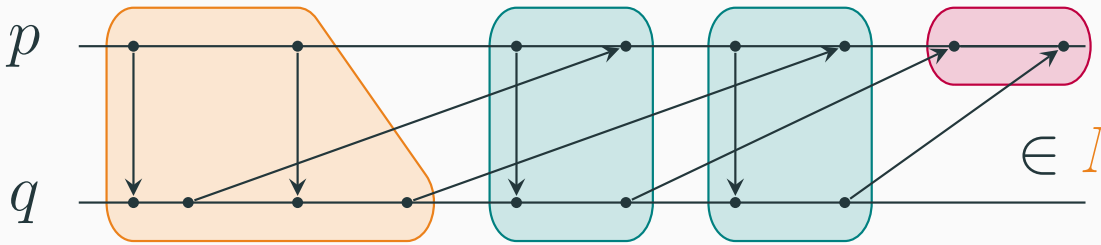
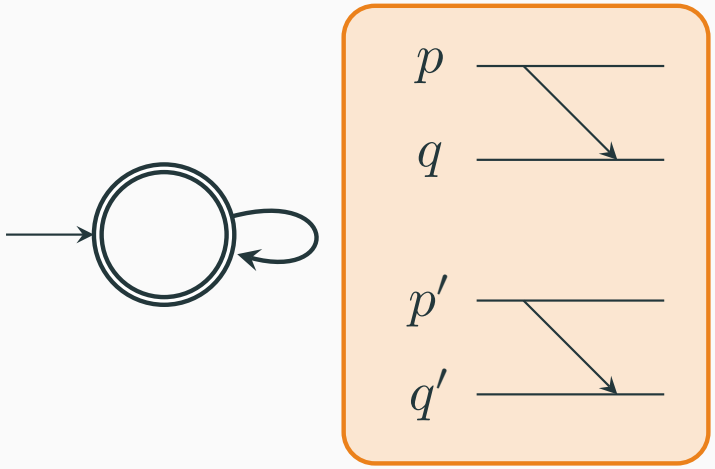




$\in M_1 \cdot M_2 \cdot M_2 \cdot M_3 \subseteq L(\mathcal{H})$

HMSCs are “too” expressive

$\mathcal{H} =$



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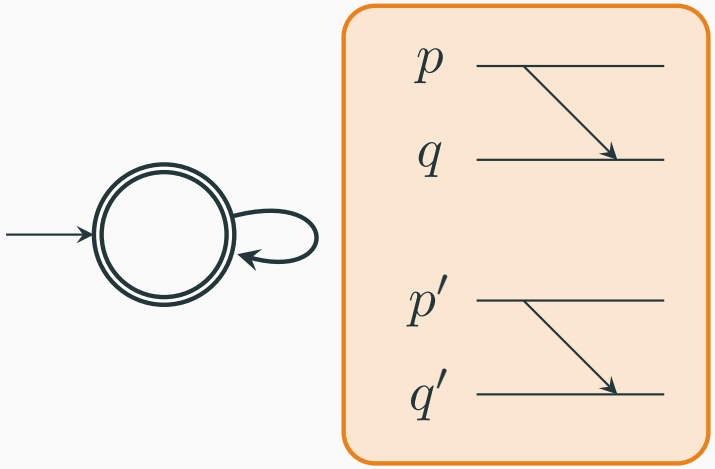

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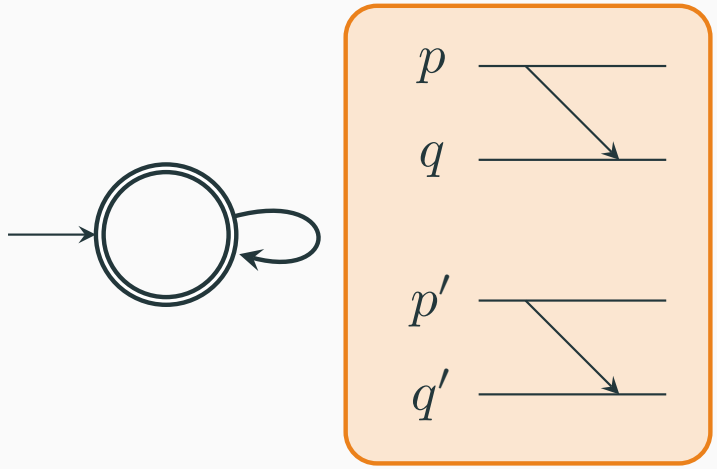
$\mathcal{H} =$ 

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$\rightarrow$ We need restrictions!

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- A cMSC M is connected if the undirected graph $(\text{events}(M), \leq \cup \leq^{-1})$ is connected.
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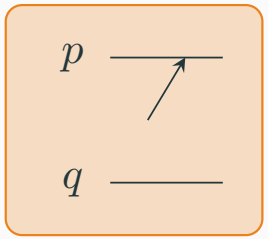
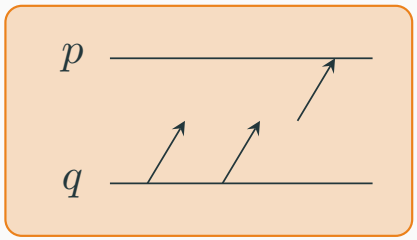
e.g., 

is both connected and weakly connected, while



is weakly connected but not connected.

Subclasses of HMSCs

- $\mathcal{H}$ is (weakly) loop-connected if for all loops in $\mathcal{H}$ with label $M_1 \cdots M_n$, every cMSC in the product $M_1 \circ \cdots \circ M_n$ is connected.

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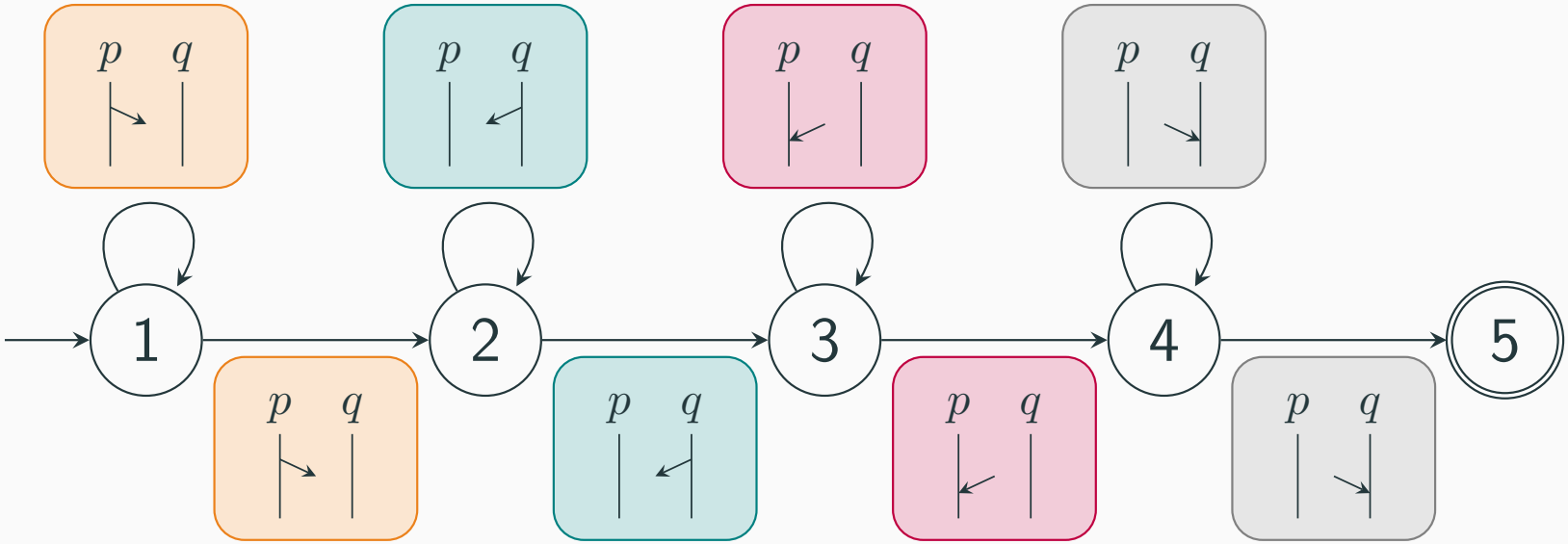
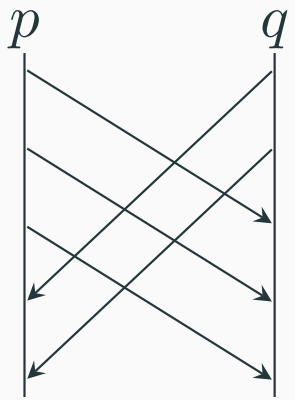
What about HMSCs that are connected but not safe?

Example



$L(\mathcal{H})$ is not $\exists B$-bounded, but is recognisable by a CFM.



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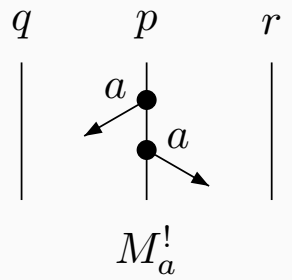
Theorem (Bollig, F., Gastin 2025)

- Emptiness of unrestricted HMSCs is undecidable, even with a message alphabet of size one.
- With a message alphabet of size at least two, emtpiness of HMSCs is **undecidable** even for **loop-connected HMSCs**. This is true even with only two processes, or three processes and flat HSMCs.

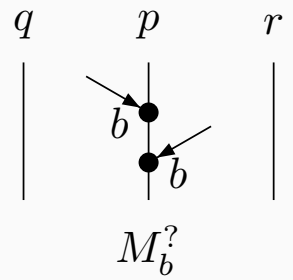
Undecidability for loop-connected HMSCs

Reduction from Post Correspondance Problem:

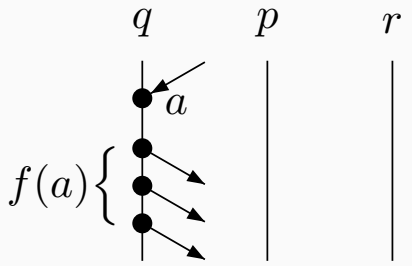
$f, g : A^* \to B^*$



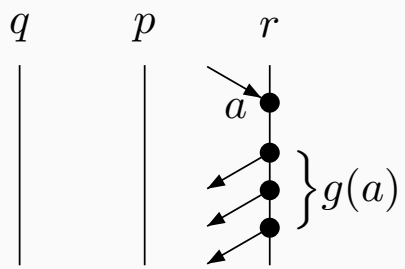
$M_a^!$



$M_b^?$



M_a^f

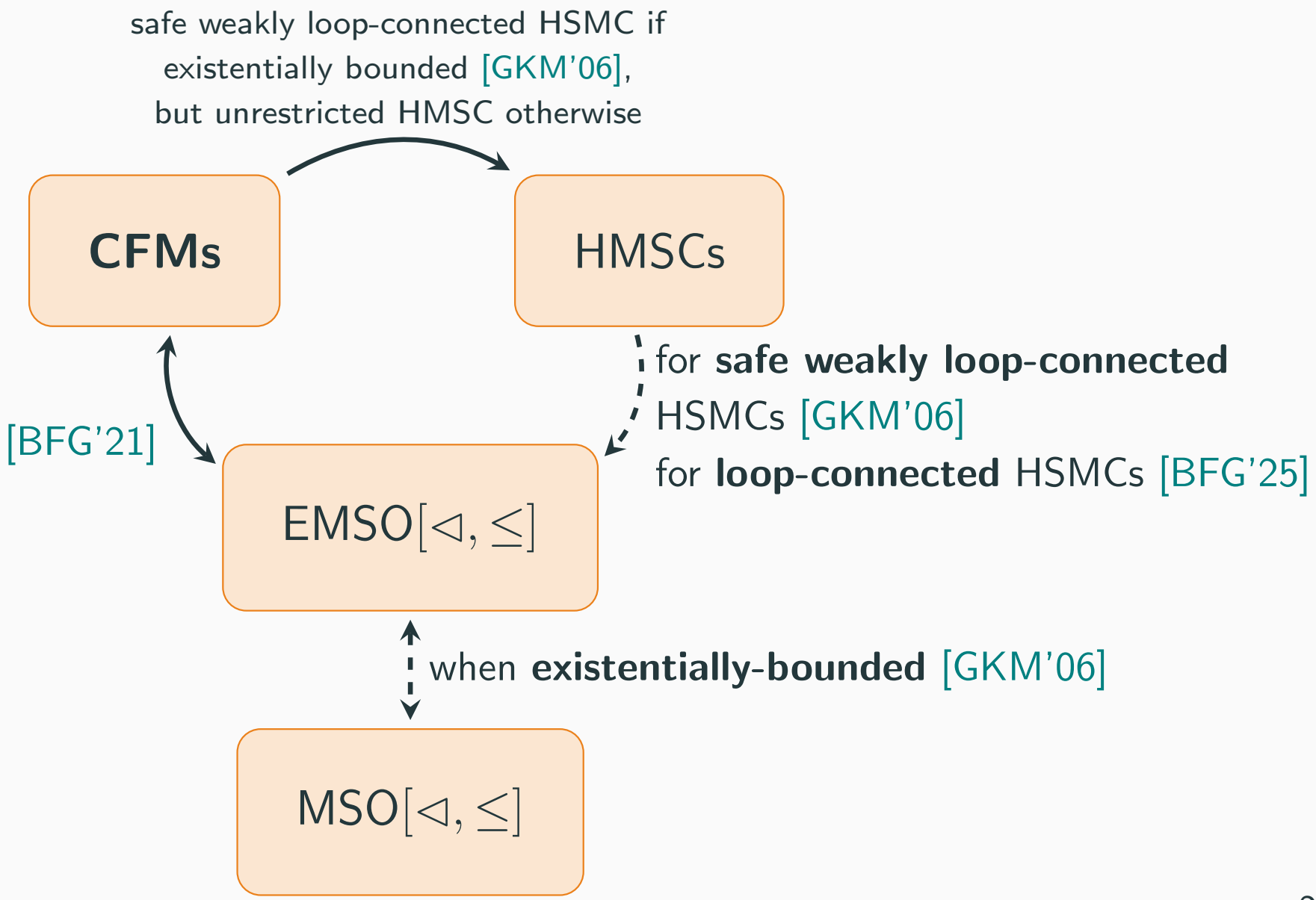


M_a^g

There exists $u \in A^+$ such that $f(u) = g(u)$ if and only if

$$\Big(\sum_{a\in A} M_a^!\Big)^+ \cdot \Big(\sum_{a\in A} M_a^f\Big)^+ \cdot \Big(\sum_{a\in A} M_a^g\Big)^+ \cdot \Big(\sum_{b\in B} M_b^?\Big)^+ \neq \emptyset$$

Conclusion


safe weakly loop-connected HSMC if
existentially bounded [GKM'06],
but unrestricted HMSC otherwise
CFMs
HMSCs
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for **safe weakly loop-connected**
HSMCs [GKM'06]
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